Mathematics 1120H – Calculus II: Integrals and Series
TRENT UNIVERSITY, Summer 2018
Final Examination
19:00-22:00 on Monday, 30 July, in CGS 105.

Time: *3 hours.* *Brought to you by* Стефан Біланюк.

Instructions: Do parts **A**, **B**, and **C**, and, if you wish, part **D**. Show all your work and justify all your answers. *If in doubt about something,* **ask!**

Aids: Any calculator; (all sides of) one aid sheet; one (1) brain (no neuron limit).

Part A. Do all four (4) of **1**–**4**.

1. Evaluate any *four* (4) of the integrals **a**–**f**. *[20 = 4 × 5 each]*

a. $\displaystyle\int_{-1}^{0} \frac{1}{x^2+2x+2}\,dx$ **b.** $\displaystyle\int_{0}^{1} \frac{1}{\sqrt{y}}\,dy$ **c.** $\displaystyle\int_{-\pi/4}^{\pi/4} \sec^2(z)\tan(z)\,dz$

d. $\displaystyle\int \left(1+w^2\right)^{1/2}\,dw$ **e.** $\displaystyle\int_{0}^{\infty} ve^{-v}\,dv$ **f.** $\displaystyle\int \frac{u+1}{u^3-u}\,du$

2. Determine whether the series converges in any *four* (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{3^n}$ **b.** $\displaystyle\sum_{m=1}^{\infty} \frac{(-1)^m}{\sqrt{m!}}$ **c.** $\displaystyle\sum_{\ell=2}^{\infty} \frac{\ell+2}{\ell^{5/2}+\ell^{3/2}+\ell^{1/2}}$

d. $\displaystyle\sum_{k=3}^{\infty} \frac{3}{k\left[\ln(k)\right]^2}$ **e.** $\displaystyle\sum_{j=4}^{\infty} \frac{j\cos(j)}{(2j)!}$ **f.** $\displaystyle\sum_{i=5}^{\infty} e^{-i}\arctan(i)$

3. Do any *four* (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. Find the Taylor series at $a=0$ of $f(x)=\dfrac{1}{x+1}$.

b. Find the arc-length of the curve $y=\ln\left(\cos(x)\right)$, where $0 \le x \le \dfrac{\pi}{4}$.

c. Suppose $a_0=a_1=1$ and $a_n=a_{n-1}+a_{n-2}$ for all $n \ge 2$. Compute $\displaystyle\lim_{n\to\infty}\frac{1}{a_n}$.

d. Find the area of the region between $y=1$ and $y=e^{-x}$ for $0 \le x \le \ln(2)$.

e. Determine the radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty}\frac{2^{n+1}x^n}{4^n+1}$.

f. Use the Right-Hand Rule or the Trapezoid Rule to approximate the definite integral $\displaystyle\int_0^1 \sin(\pi x)\,dx$ to within 1 of the exact value.

4. Consider the finite region bounded by $x=0$, $y=1$, and $y=x^3$.

a. Find the area of this region. *[4]*

b. Find the volume of the solid obtained by revolving the region about $x=0$. *[8]*

Part B. Do either *one* (1) of **5** or **6**. *[14]*

5. A solid is obtained by revolving the triangle with vertices at $(1, 0)$, $(2, 0)$, and $(2, 1)$ about the y-axis..

a. Find the volume of the solid. *[7]*

b. Find the surface area of the solid. *[7]*

6. Consider the region below $y = x - 1$ and above $y = (x - 1)^2$. Find the volume of the solid obtained by revolving this region about ...

a. ... the x-axis. *[7]*

b. ... the y-axis. *[7]*

Part C. Do either *one* (1) of **7** or **8**. *[14]*

7. Use Taylor's formula to find the Taylor series at $a = 0$ of $f(x) = e^{x+1}$ and determine its radius and interval of convergence.

8. Find the Taylor series at $a = 0$ of $f(x) = \dfrac{x}{1 + x^2}$ and determine its radius and interval of convergence.

[Total = 100]

Part D. Bonus problems! If you feel like it and have the time, do one or both of these.

$\bigvee$. The longest straight line that can be drawn entirely on the surface of a perfectly flat and circular road of some constant width is 50 m long. What is the surface area of the road? *[1]*

$\bigwedge$. Write a haiku (or several :-) touching on calculus or mathematics in general. *[1]*

What is a haiku?

seventeen in three:
five and seven and five of
syllables in lines

ENJOY THE REST OF YOUR SUMMER!