Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Summer 2025 (S62)

Quiz #1 – Limits
Due on Thursday, 19 June.

NOTE. Please show all the steps in your solutions to both of the problems below.

1. Use the ε–δ definition of limits to verify that $\lim_{x\to -1}(2x+4)=2$. *[2.5]*

NOTE. You may use the standard version of the definition or the alternate game form of the definition, as you prefer.

SOLUTION. We'll use the standard ε–δ definition of limits to verify that $\lim_{x\to -1}(2x+4)=2$. This means that we need to check that for any $\varepsilon>0$ we can find a corresponding $\delta>0$ such that whenever $|x-(-1)|<\delta$, we are guaranteed to have $|(2x+4)-2|<\varepsilon$.

As usual, we find δ by working backwards from the desired conclusion:

$$\begin{aligned}|(2x+4)-2|<\varepsilon &\iff |2x+2|<\varepsilon\\ &\iff |2(x+1)|<\varepsilon\\ &\iff 2|x+1|<\varepsilon\\ &\iff 2\,|x-(-1)|<\varepsilon\\ &\iff |x-(-1)|<c\end{aligned}$$

We claim that setting $\delta=\frac{\varepsilon}{2}$ does. If $|x-(-1)|<\delta=\frac{\varepsilon}{2}$, we trace the chain of reasoning above backwards – since every step in the process is fully reversible – to get $|(2x+4)-2|<\varepsilon$.

Thus $\lim_{x\to -1}(2x+4)=2$ by the standard ε–δ definition of limits. □

2. Work out $\lim_{t\to 1}\frac{\sqrt{t}-1}{t^2-1}$ using the limit laws. *[2.5]*

Hint: Algebra!

SOLUTION. Witness the power of fully operational algebra! Note that every t close to 1 is positive, and hence $\sqrt{t}$ is well-defined and $\left(\sqrt{t}\right)^2=t$.

$$\begin{aligned}\lim_{t\to 1}\frac{\sqrt{t}-1}{t^2-1} &= \lim_{t\to 1}\frac{\sqrt{t}-1}{(t-1)(t+1)} = \lim_{t\to 1}\frac{\sqrt{t}-1}{\left(\left(\sqrt{t}\right)^2-1\right)(t+1)}\\ &= \lim_{t\to 1}\frac{\sqrt{t}-1}{\left(\sqrt{t}-1\right)\left(\sqrt{t}+1\right)(t+1)}\\ &= \lim_{t\to 1}\frac{1}{\left(\sqrt{t}+1\right)(t+1)} = \frac{1}{\left(\sqrt{1}+1\right)(1+1)}\\ &= \frac{1}{(1+1)\,2} = \frac{1}{2\cdot 2} = \frac{1}{4} \quad \blacksquare\end{aligned}$$