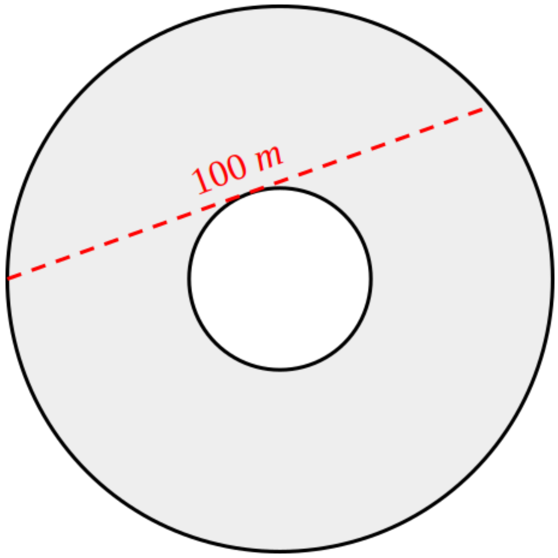
Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
Trent University, Summer 2025 (S62)

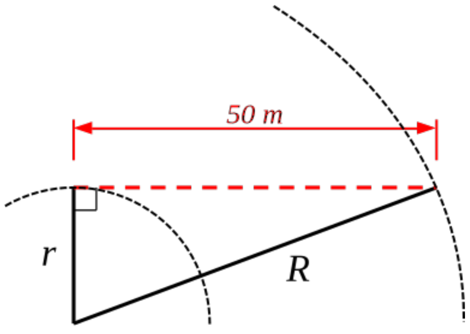
Quiz #0 – Around and Around We Go
Due on Tuesday, 17 June.

1. The paved portion of a racetrack has the shape of an annulus, a region between two circles that have a common centre. The longest possible straight line that fits entirely into this paved portion is 100 m long. Find the area of the paved portion of the racetrack and explain your reasoning as completely as you can. *[5]*



Hint: A line tangent to a circle at a given point is perpendicular to the radius of the circle at that point.

Solution. Pick a 100 m line that just fits inside the track. It has two endpoints, which are both on the larger circle, and touches the smaller circle at it's midpoint. (Why?) Connect the centre of the circles to one of the endpoints and to the midpoint, as in the diagram below.



Eaxh of these connections is a radius: the connection to the endpoint is a radius, call it R, of the larger circle, and the connection to the midpoint is a radius, call it r, of the smaller circle. Since the line touches the smaller circle only at its midpoint, it is tangent to the circle, and so is perpendicular to the radius of the smaller circle. This means we have created a right triangle with short sides of length r and 50, respectively, and a hypotenuse of length R. By the Pythagorean Theorem, we therefore have $r * 2 + 50^2 = R^2$, which we can rearrange to give $R^2 - r^2 = 50^2$.

Since the area of a circle of radius s is πs^2, the area of the paved portion of the track is given by the difference of the areas of the two circles:

$$\text{Area} = \pi R^2 - \pi r^2 = \pi\left(R^2 - r^2\right) = \pi 50^2 = 2500\pi\ m^2 \quad \blacksquare$$