Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals

TRENT UNIVERSITY, Summer 2025 (S62)

Assignment #1 – Solving With SageMath

Due on Friday, 27 June.[*]

In all that follows, let

$$f(x) = \ln\left(x + \sqrt{x^2+1}\right).$$

Most of this assignment is concerned with finding $f^{-1}(x)$, the inverse function of $f(x)$, *i.e.* the function such that $y = f(x) \iff x = f^{-1}(y)$, or $f^{-1}(f(x)) = x$, at least when everything is duly defined.

Please recall that SageMath calls the natural logarithm function `log`. Also, take a peek at Section 3.4 of *Sage for Undergraduates*, by Gregory Bard, which has to do have to do with plotting graphs of implicitly defined curves, before doing question **1c**, and Section 1.8, which has to do with solving equations, before doing question **2**.

1. **a.** What is the domain of $f(x)$? *[0.5]*

b. Use SageMath to plot $y = f(x)$ for $-10 \le x \le 10$ and $-3 \le y \le 3$. *[1]*

c. Use SageMath to plot $y = f^{-1}(x)$, otherwise known as $x = f(y)$, for $-3 \le x \le 3$ and $-10 \le y \le 10$. *[2]*

2. **a.** Ask SageMath to solve the equation $x = f(y)$ for y. *[1]*

SageMath will probably give you a solution with a square root in which y occurs ...

b. Rearrange the solution SageMath gave you in part **a** to eliminate the square root and ask SageMath to solve the modified equation for y. *[2]*

This time SageMath should give you an answer for y in terms of x only.

c. What is $f^{-1}(x)$ in terms of x? What is the domain of $f^{-1}(x)$? *[1]*

3. Work out what $f^{-1}(x)$ is in terms of x by hand, showing all the steps. *[2.5]*

On Problems

Problems worthy
of attack,
Prove their worth
by hitting back.

Another grook by Piet Hein.

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[*] You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper. You may work together and look things up, so long as you write up your submission by yourself and give due credit to your collaborators and any sources you actually used.