

Mathematics 1110H – Calculus I: Limits, derivatives, and Integrals

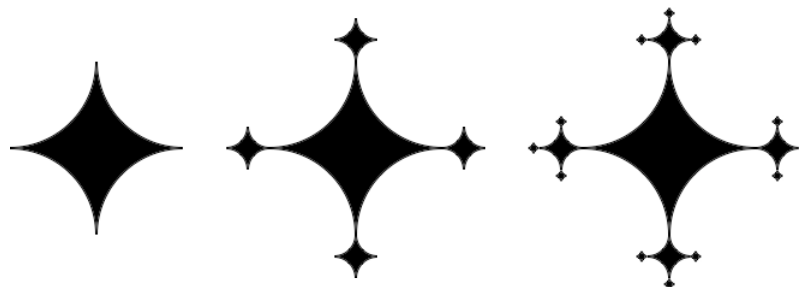
TRENT UNIVERSITY, Summer 2018

Assignment #1

Square - Circle = Squircle?

Due on Monday, 14 May.

Let's call the shape that you get by removing four mutually tangent quarter-circles with radius $\frac{s}{2}$ from a square with side length s a *squircle** of side s . (See the leftmost shape in the diagram below.)



1. What are the area and perimeter of a squircle of side s ? [1]

A single squircle has four points where the quarter-circles that were removed met. Consider the following process:

At step $n = 0$ we have a single squircle for which $s = 2$.

At step $n = 1$, we attach four squircles for which $s = \frac{1}{4} \cdot 2 = \frac{1}{2}$ to the squircle in step 0, attaching one (at one of its points) to each point of the larger squircle. (See the middle shape in the diagram above.) The resulting shape has $3 \cdot 4 = 12$ points (where quarter-circles met) to which nothing is yet attached. Let's call these the *free* points of the shape.

At step $n = 2$, we attach a squircle for which $s = \frac{1}{4} \cdot \frac{1}{2} = \left(\frac{1}{4}\right)^2 \cdot 2 = \frac{1}{8}$ to each of the free points in the shape in step 1. (See the rightmost shape in the diagram above.) The resulting shape has $3 \cdot 12 = 3 \cdot (3 \cdot 4) = 3^2 \cdot 4 = 36$ free points.

At step $n = 3$, we attach a squircle for which $s = \frac{1}{4} \cdot \frac{1}{8} = \left(\frac{1}{4}\right)^3 \cdot 2 = \frac{1}{32}$ to each of these the free points in the shape in step 2. (Draw your own picture!) The resulting shape has $3 \cdot 36 = 3 \cdot (3^2 \cdot 4) = 3^3 \cdot 4 = 108$ free points.

Repeat for each integer $n > 3 \dots$

2. Find formulas for the values of s for the squircles added at step n and for the number of free points of the shape obtained in step n . [2]
3. Find a formula for the total length of the perimeter of the shape obtained in step n . [2]
4. Find a formula for the total area of the shape obtained in step n . [2]
5. What are the total length of the perimeter and the total area of the shape obtained after infinitely many steps of the process? [3]

* This shape probably has a name already, but I don't know it ... :-)