Mathematics 1100Y – Calculus I: Calculus of one variable
Trent University, Summer 2012

Solutions to Assignment #9
It's a mad, mad world!

In a certain part of Wonderland, all creatures are either mad, in which case they believe exactly those things which are false, or sane, in which case they believe exactly those things which are true. The Duchess and Alice discuss problems about such creatures below...

The caterpillar and the lizard.

"Well," replied the Duchess, "take, for example, the Caterpillar and Bill the Lizard. The Caterpillar believes that both of them are mad."

"Which of them is really mad?" asked Alice.

"I shouldn't have to tell you *that*!" replied the Duchess. "I have given you enough information for you to deduce the answer."

1. Is the Caterpillar mad or sane? And what about the Lizard? Explain your reasoning! *[5]*

SOLUTION. Here is the solution given by Raymond Smullyan himself in *Alice in Puzzle-Land*:

> The Caterpillar believes that he and the Lizard are both mad. If the Caterpillar were sane, it would be false that he and the Lizard were both mad, hence the Caterpillar (being sane) couldn't possibly believe this false fact. Therefore the Caterpillar must be mad. Since he is mad, his belief is wrong, so it is not true that *both* are mad. So the other one (the Lizard) must be sane. Therefore the Carerpillar is mad and the Lizard is sane. ∎

What about these three?

"I've always wondered about the March Hare, the Hatter, and the Dormouse," said Alice. "The Hatter is *called* the Mad Hatter, but is he really mad? And what about the March Hare and the Dormouse?"

"Well," replied the Duchess, " the Hatter once expressed the belief that the March Hare does not believe that all of them are sane. Also, the Dormouse believes that the March Hare is sane."

2. What can you deduce about these three? Explain your reasoning! *[5]*

SOLUTION. Here is the solution given by Raymond Smullyan himself in *Alice in Puzzle-Land*:

> Suppose the Hatter is sane. Then his belief is correct, which means that the March Hare does not believe that all three are sane. Then the March Hare must be sane, because if he were mad, he would believe the false proposition that all three are sane. Then the Dormouse, believing that the March Hare is sane, must be sane, which makes all three sane. But then hown could the sane March

Hare fail to believe the trueproposition that all three are sane? Therefore it is contradictory to assume the Hatter sane; the Hatter must really be mad.

Since the Hatter is mad, his belief is wrong, and therefore the March Hare does believe that all three are sane. Of course the March Hare is wrong (since the Hatter is not sane), and so the March Hare is also mad. Then the Dormouse, believing the Narch Hare is sane, is also mad, and so all three are mad (which is not too surprising!).

NOTE. These problems were stolen from — er, creative inspiration was provided by — Raymond Smullyan. If you like this sort of thing, check out his books *Alice in Puzzle-Land* and *What is the name of this book?*