Mathematics 1100Y – Calculus I: Calculus of one variable
TRENT UNIVERSITY, Summer 2012
Final Examination

Time: *14:00–17:00, on Tuesday, 7 August, 2012.* *Brought to you by* Стефан Біланюк.

Instructions: Do parts ♡, ♢, and ♣, and, if you wish, part ♠. Show all your work and justify all your answers. *If in doubt about something,* **ask!**

Aids: Calculator; up to two (≤ 2) aid sheets; at most one (≤ 1) brain.

Part ♡. Do all four (4) of **1**–**4**.

1. Compute $\dfrac{dy}{dx}$ as best you can in any *three* (3) of **a**–**f**. *[15 = 3 × 5 each]*

a. $y = \tan(2x)$ **b.** $e^x e^y = 1$ **c.** $y = e^x \cos(x)$

d. $y = \dfrac{x^2+9}{x+2}$ **e.** $\begin{array}{c} y = t+1 \\ x = \sec(t) \end{array}$ **f.** $y = \displaystyle\int_1^x e^{z+1}\, dz$

2. Evaluate any *three* (3) of the integrals **a**–**f**. *[15 = 3 × 5 each]*

a. $\displaystyle\int \frac{1}{x^3+4x}\, dx$ **b.** $\displaystyle\int_e^\infty \frac{1}{x\ln(x)}\, dx$ **c.** $\displaystyle\int \cos(2t+1)\, dt$

d. $\displaystyle\int_0^{\pi/2} \sin^2(z)\cos^3(z)\, dz$ **e.** $\displaystyle\int e^x \sec\left(e^x\right)\, dx$ **f.** $\displaystyle\int_0^1 \arctan(x)\, dx$

3. Do any *three* (3) of **a**–**f**. *[15 = 3 × 5 each]*

a. Use the Right-hand Rule to compute the definite integral $\displaystyle\int_0^2 (x+1)\, dx$.

b. Compute $\displaystyle\lim_{n\to\infty} n\sin(n\pi)$.

c. Sketch the region between $r = 0$ and $r = \sec(\theta)$, for $0 \leq \theta \leq \pi/4$, in polar coordinates and find its area.

d. Find the area of the surface obtained by revolving the curve $y = x$, for $0 \leq x \leq 1$, about the y-axis.

e. Use the limit definition of the derivative to compute $f'(2)$ if $f(x) = x^2 + 1$.

f. Determine whether the series $\displaystyle\sum_{n=0}^{\infty} \frac{n}{e^{2n}}$ converges or diverges.

4. Consider the curve $y = \dfrac{x^2}{2}$ $0 \leq x \leq 2$.

a. Sketch this curve. *[1]*

b. Sketch the surface obtained by revolving this curve about the x-axis. *[1]*

c. Compute either *i.* the length of the curve or *ii.* the area of this surface. (Not both!) *[8]*

Part $\diamondsuit$. Do any *two* (2) of **5**–**7**. *[30 = 2 × 15 each]*

5. Sketch the solid obtained by revolving the region below $y = \sqrt{25 - x^2}$ and above $y = 0$, for $4 \leq x \leq 5$, about the y-axis and find its volume. *[15]*

6. Find the domain, all the intercepts, maximum, minimum, and inflection points, and all the vertical and horizontal asymptotes of $f(x) = xe^x$, and sketch its graph. *[15]*

7. Freyja and Hretha sprint 100 m in lanes that are 5 m apart. The two start simultaneously at $t = 0$ s. Freyja runs at 9.6 m/s and Hretha at $10\,m/s$.

- **a.** How far ahead is Hretha when she crosses the finish line? When does Freyja cross the finish line? *[1]*
- **b.** Determine how quickly Hretha is pulling ahead as she crosses the finish line. *[1]*
- **c.** Determine how the distance [along a direct line] between the two is changing at the instant that Hretha crosses the finish line. *[8]*
- **d.** The two runners' starting positions and their positions at any instant thereafter form a trapezoid. How is the area of this trapezoid changing at the instant that Hretha crosses the finish line? *[5]*

Part $\clubsuit$. Do *one* (1) of **8** or **9**. *[15 = 1 × 15 each]*

8. Consider the power series $\displaystyle\sum_{n=0}^{\infty} \frac{n+1}{2^{n+1}} x^n$.

- **a.** Find the radius of convergence of this power series. *[10]*
- **b.** What function has this power series as its Taylor series at 0? *[5]*

9. Let $f(x) = x\sin(3x)$.

- **a.** Find the Taylor series at 0 of $f(x)$. *[10]*
- **b.** Determine the radius of convergence of this Taylor series. *[5]*

[Total = 100]

Part $\spadesuit$. Bonus problems! Do them (or not), if you feel like it.

0. Sketch the graph of $r = 1 - e^{-\theta}$ [polar coordinates!] for $\theta \geq 0$, and explain why it has the shape it does. *[2]*

−1. Write an original poem touching on calculus or mathematics in general. *[2]*

I THE COURSE WAS FUN, AT LEAST A LITTLE.
ENJOY THE REST OF THE SUMMER!