

Mathematics 1100Y – Calculus I: Calculus of one variable

TRENT UNIVERSITY, SUMMER 2011

Solutions to Assignment #2

Plotting with Maple[†]

Before attempting the questions below, please read the handout *A very quick start with Maple* and play around with Maple a little. It might also be useful to skim though *Getting started with Maple 10* by Gilberto E. Urroz – read those parts concerned with plotting curves more closely! – and perhaps keep it handy as a reference. You can find links to both documents on the MATH 1100Y web page. Maple's help facility may also come in handy, especially when trying to make out the intricacies of what the `plot` command and its options and variations do.

1. Use Maple to plot the curves $y = 1 - x^2$, $-1 \leq x \leq 1$, and $x = 1 - y^2$, $-1 \leq y \leq 1$.
[Please submit a printout of your worksheet(s).] [2]

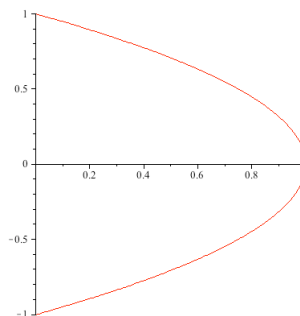
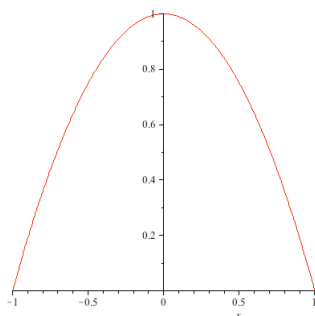
SOLUTION. Here are the graphs generated by the Maple commands

```
> plot(1-x^2,x=-1..1);
```

and

```
> plot([1-t^2,t,t=-1..1]);
```

respectively.



Note that the second graph was obtained by setting up $x = 1 - y^2$, $-1 \leq y \leq 1$, as a parametric curve. It could also have been obtained using the `implicitplot` command. This, however, requires first telling Maple to add the command to the default environment:

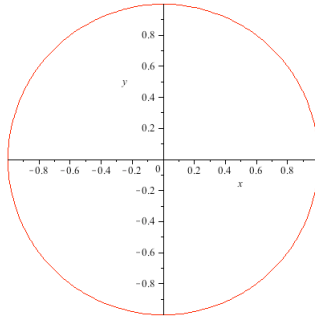
```
> with(plots,implicitplot);  
> implicitplot(x=1-y^2,x=-1..1,y=-1..1);     □
```

[†] ... though it may sometimes feel as if Maple is plotting *against* you.

2. Use Maple to plot the curve $x^2 + y^2 = 1$. Please submit a printout of your worksheet as your solution. [1]

SOLUTION. Here is the graph generated by the Maple commands

```
> with(plots,implicitplot);  
> implicitplot(x^2+y^2=1,x=-1..1,y=-1..1);
```



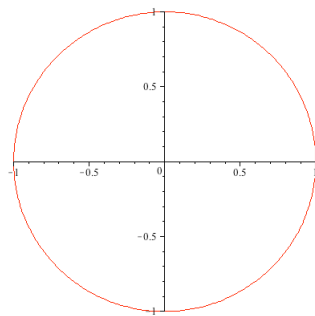
To see how to set this up as a parametric curve, see 3. □

One way to describe or define a curve in two dimensions is by way of *parametric equations*, $x = f(t)$ and $y = g(t)$, where the x and y coordinates of points on the curve are simultaneously specified by plugging a third variable, called the *parameter* (in this case t), into functions $f(t)$ and $g(t)$. This approach can come in handy for situations where it is impossible to describe all of a curve as the graph of a function of x (or of y) and arises pretty naturally in various physics problems. (Think of specifying, say, the position (x, y) of a moving particle at time t .) We will see more of parametric curves later on in Chapter 10 of the textbook.

3. Use Maple to plot the parametric curve $x = \cos(t)$, $y = \sin(t)$, where $0 \leq t \leq 2\pi$. [Please submit a printout of your worksheet.] [1.5]

SOLUTION. Here is the graph generated by the Maple command

```
> plot([cos(t),sin(t),t=0..2*Pi]);
```



□

4. The graph you obtained for **3** should look just like one of the graphs you obtained in **1** or **2**. Which one? Why are these graphs identical? [1.5]

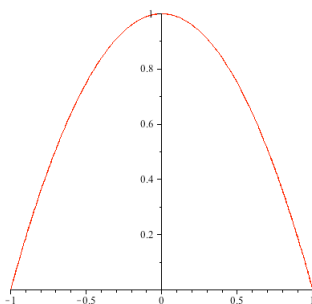
SOLUTION. The graph obtained in the solution to **3** is (obviously!) just like the graph obtained in the solution to **2**. The reason isn't hard to figure out:

Suppose $x = \cos(t)$ and $y = \sin(t)$ are the coordinates of a point on the parametric curve in **3**. Then $x^2 + y^2 = \cos^2(t) + \sin^2(t) = 1$, so it is also a point on the curve $x^2 + y^2 = 1$. Conversely, if (x, y) is a point on the curve $x^2 + y^2 = 1$, then $x = \cos(t)$ and $y = \sin(t)$ for the counterclockwise angle t that the line segment from $(0, 0)$ to (x, y) makes with the positive x -axis. (Note that allowing t to vary from 0 to 2π lets us capture every point all around the circle in this way.) Hence the two curves include exactly the same points, *i.e.* they give the same graph. \square

5. Use **Maple** to plot the parametric curve $x = \sin(t)$, $y = \frac{1}{2} + \frac{1}{2} \cos(2t)$, where $0 \leq t \leq 2\pi$. [Please submit a printout of your worksheet.] [1.5]

SOLUTION. Here is the graph generated by the **Maple** command

```
> plot([sin(t), 1/2+(1/2)*cos(2*t)], t=0..2*Pi);
```



\square

6. The graph you obtained for **5** should look just like one of the graphs you obtained in **1** or **2**. Which one? Why are these graphs identical? [2.5]

SOLUTION. Suppose (x, y) is on the parametric curve, *i.e.* $x = \sin(t)$ and $y = \frac{1}{2} + \frac{1}{2} \cos(2t)$. We will use one form of the double-angle formula for \cos here, namely $\cos(2t) = 1 - 2\sin^2(t)$. Thus

$$\begin{aligned} y &= \frac{1}{2} + \frac{1}{2} \cos(2t) = \frac{1}{2} + \frac{1}{2} (1 - 2\sin^2(t)) \\ &= \frac{1}{2} + \frac{1}{2} (1 - 2x^2) = \frac{1}{2} + \frac{1}{2} - \frac{1}{2} 2x^2 = 1 - x^2, \end{aligned}$$

so (x, y) is on the curve $y = 1 - x^2$.

Conversely, suppose (x, y) is on the curve $y = 1 - x^2$. I will leave it to you to figure out how to find the angle t , with $0 \leq t \leq 2\pi$, so that $x = \sin(t)$ and $y = \frac{1}{2} + \frac{1}{2} \cos(2t)$.[‡] \square

[‡] $t = \arctan\left(\frac{x}{1-x^2}\right)$, more or less, but only more or less ...

REFERENCES

1. *A very quick start with Maple*, by Stefan Bilaniuk, which can found (pdf) at:
<http://www.trentu.ca/mathematics/sb/1100Y/Summer-2011/MATH1100Y-maple-start.pdf>
2. *Getting started with Maple 10*, by Gilberto E. Urroz (2005), which can found (pdf) at: <http://www.trentu.ca/mathematics/sb/1100Y/Summer-2011/GettingStartedMaple10.pdf>