Mathematics 1100Y – Calculus I: Calculus of one variable
TRENT UNIVERSITY, Summer 2011
Final Examination

Time: 09:00–12:00, on Wednesday, 3 August, 2011. *Brought to you by* Стефан.
Instructions: Show all your work and justify all your answers. *If in doubt,* **ask!**
Aids: Calculator; two (2) aid sheets; one (1) brain [may be caffeinated].

Part I. Do *all* three (3) of **1**–**3**.

1. Compute $\frac{dy}{dx}$ as best you can in any *three* (3) of **a**–**f**. *[15 = 3 × 5 each]*

a. $x = e^{x+y}$ **b.** $y = \int_0^{-x} te^t\,dt$ **c.** $y = x^2\ln(x)$

d. $y = \frac{x}{\cos(x)}$ **e.** $y = \sec^2(\arctan(x))$ **f.** $y = \sin(e^x)$

2. Evaluate any *three* (3) of the integrals **a**–**f**. *[15 = 3 × 5 each]*

a. $\int \frac{2x}{\sqrt{4-x^2}}\,dx$ **b.** $\int_0^{\pi/2} \sin(z)\cos(z)\,dz$ **c.** $\int x^2\ln(x)\,dx$

d. $\int_{-\infty}^{\ln(3)} e^s\,ds$ **e.** $\int \frac{1}{\sqrt{1+x^2}}\,dx$ **f.** $\int_1^2 \frac{1}{w^2+w}\,dw$

3. Do any *five* (5) of **a**–**i**. *[25 = 5 × 5 ea.]*

a. Determine whether the series $\sum_{n=2}^{\infty} \frac{(-1)^n n^2}{3^n}$ converges absolutely, converges conditionally, or diverges.

b. Why must the arc-length of $y = \arctan(x)$, $0 \le x \le 13$, be less than $13 + \frac{\pi}{2}$?

c. Find a power series equal to $f(x) = \frac{x}{1+x}$ (when the series converges) without using Taylor's formula.

d. Find the area of the region between the origin and the polar curve $r = \frac{\pi}{2} + \theta$, where $0 \le \theta \le \pi$.

e. Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{n}{2^n} x^n$.

f. Use the limit definition of the derivative to compute $f'(0)$ for $f(x) = 2x - 1$.

g. Compute the area of the surface obtained by rotating the the curve $y = \frac{x^2}{2}$, where $0 \le x \le \sqrt{3}$, about the y-axis.

h. Use the Right-hand Rule to compute the definite integral $\int_0^3 (x+1)\,dx$.

i. Use the $\varepsilon - \delta$ definition of limits to verify that $\lim_{x\to 2}(x+1) = 3$.

Part II. Do any *three* (3) of **4–8**.

4. Find the domain, all maximum, minimum, and inflection points, and all vertical and horizontal asymptotes of $f(x) = \dfrac{x^2}{x^2+1}$, and sketch its graph. *[15]*

5. Do both of **a** and **b**.

a. Verify that $\displaystyle\int \sqrt{x^2-1}\,dx = \frac{1}{2}x\sqrt{x^2-1} - \frac{1}{2}\ln\left(x+\sqrt{x^2-1}\right) + C$. *[7]*

b. Find the arc-length of $y = \dfrac{1}{2}x\sqrt{x^2-1} - \dfrac{1}{2}\ln\left(x+\sqrt{x^2-1}\right)$ for $1 \le x \le 3$. *[8]*

6. Sketch the solid obtained by rotating the square with corners at $(1,0)$, $(1,1)$, $(2,0)$, and $(2,1)$ about the y-axis and find its volume and surface area. *[15]*

7. Do all three (3) of **a**–**c**.

a. Use Taylor's formula to find the Taylor series at 0 of $f(x) = \ln(x+1)$. *[7]*

b. Determine the radius and interval of convergence of this Taylor series. *[4]*

c. Use your answer to part **a** to find the Taylor series at 0 of $\dfrac{1}{x+1}$ without using Taylor's formula. *[4]*

8. A spherical balloon is being inflated at a rate of 1 m^3/s. How is its surface area changing at the instant that its volume is 36 m^3? *[15]*
[Recall that a sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$.]

[Total = 100]

Part MMXI - Bonus problems.

13. Show that $\ln(\sec(x) - \tan(x)) = -\ln(\sec(x) + \tan(x))$. *[2]*

41. Write an original poem touching on calculus or mathematics in general. *[2]*

I hope that you had some fun with this!
Get some rest now ...