

# Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals

TRENT UNIVERSITY, Fall 2025

## Assignment #3

### A Differential Equation and an Inverse Function

*Due on Friday, 17 October.\**

You might want to skim subsection 4.22.1 of Gregory Bard's *Sage for Undergraduates*, or go to the labs, before tackling this assignment. At the very least, check out the `diff` and `desolve` commands, and `function` declarations, in Maya Peters' *Glossary of commands*.

1. Consider the *differential equation*  $\frac{1}{y^2} \cdot \frac{dy}{dx} = e^{-x} - e^x$ .
  - a. Use SageMath's `desolve` command to find the general solution to this differential equation. [2]
  - b. Use SageMath's `desolve` command to find the particular solution to this differential equation that satisfies the *initial condition*  $y = \frac{1}{2}$  when  $x = 0$ . [2]
  - c. Check by hand that the solution you obtained in **b** satisfies the given differential equation and the given initial condition. [1]

NOTE. That is, find the function  $y$  of  $x$  that satisfies this equation and, in **b**, the additional condition, too. You may need to use the `solve` command or do a bit of algebra yourself to put what SageMath first gives you into a more useful form.

2. In what follows, let  $f(x)$  be your solution to **1b** above.
  - a. Plot  $y = f(x)$  for  $-5 \leq x \leq 5$ . [1]
  - b. From your plot, what are the largest pieces of  $f(x)$  for which there is an inverse function? [1]
  - c. Use SageMath's `solve` command to find (the possibilities for)  $f^{-1}(x)$ , the inverse function of  $f(x)$ . [1]
  - d. Find (the possibilities for)  $f^{-1}(x)$  by hand, showing all the principal steps. [2]

HINT.  $y = f^{-1}(x) \iff x = f(y)$ .

[Total = 10]

#### On Problems

Our choicest plans  
    have fallen through,  
our airiest castles  
    tumbled over,  
because of lines  
    we neatly drew  
and later neatly  
    stumbled over.

*Yet another grook by Piet Hein.*

---

\* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper as soon as you can. You may work together, look things up, and use whatever tools you like, so long as you write up your submission by yourself and give due credit to your collaborators and any sources and tools you actually used.