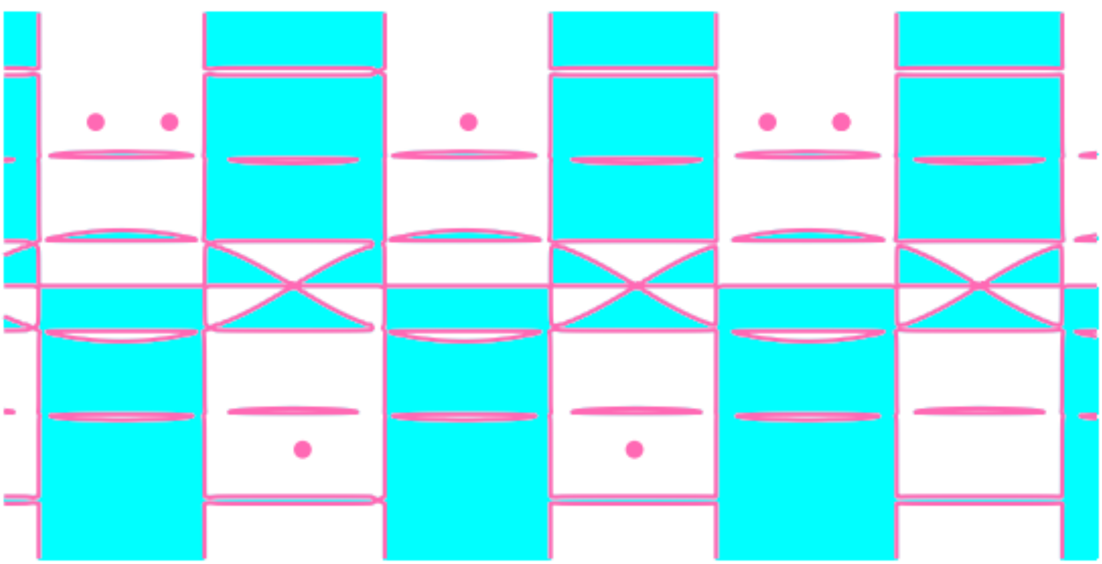
In [1]:
```
# Include these two lines of code at the beginning of every notebook you op
en.
# It will allow you to receive more than one output from a single code chun
k/cell.
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = "all"
```

MATH1110 Lab 3: Limits

Remember to run the chunk of code above as you get started!

In [77]:
```
x = var('x')
y = var('y')
func = tan(y)==y/sin(x/2) # remember this guy?! You made them in an exercis
e set last week!
Aqua = implicit_plot(func, (x, -20, 20), (y, -10, 10), color = 'hotpink', \
              fill = True, fillcolor = 'aqua', axes = False)
Barbie = points( [[14*sin(pi*x/7)-3, 6*(-1)^x] for x in range(-10, 10)], co
lor = 'hotpink', size = 40)
Barbie+Aqua
```

Out[77]:



How'd Assignment 1 go? Terribly? Not too shabby? If you're still feeling clueless, please come talk to Maya or Stefan and we'll get you up to speed!

Objectives for today:

1. Limits
2. The Solve function

1. Limits

1. Limits

Looking at a plot, we can get a good sense of potential areas of discontinuity. Limits are a more precise way we can test the behaviour of a function, such as around asymptotes, for example.

Let's use Sage to calculate some limits to find the exact values. Let's first look at $\arctan(x)^2$. Do you think this function has any asymptotes? How could we use limits to find out?

Hint: Infinite limits can be inputted with *infinity* or *oo* (two lowercase o's).

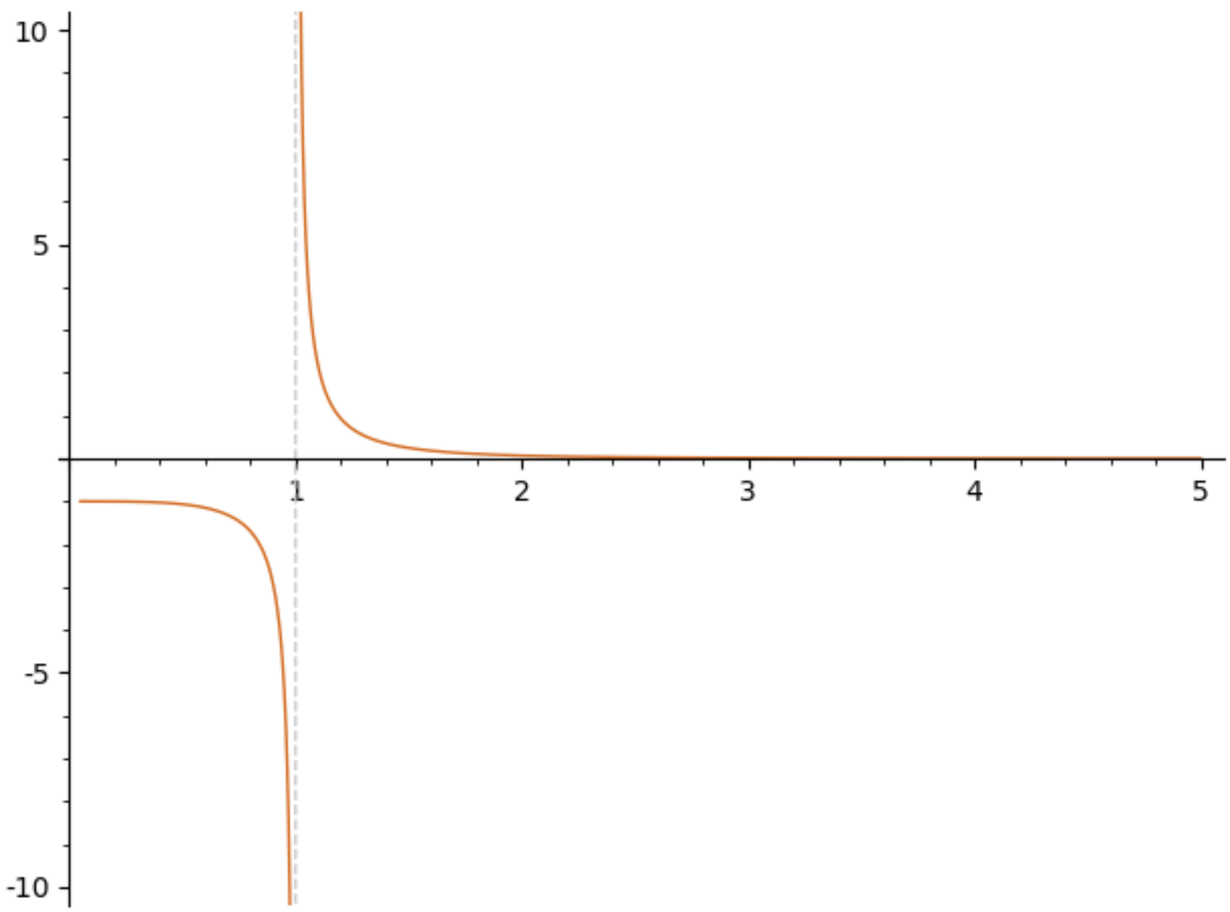
In [ ]:

Limits can also be one-sided with the *dir=* argument. Take the regular and one-sided limits as $x \to 1$, where the function is discontinuous.

In [1]:

```
y2 = 1/(x^4-1)
plot(y2, x, 0, 5, ymin = -10, ymax = 10, detect_poles = 'show', color = 'chocolate')
# remember that the detect_poles argument is how we show asymptotes in Sage
```

Out[1]:



Exercise questions

1. Is the function $\sqrt{t} - t^2 + \cos(4t)$ continuous? Make its plot, and find the limit as $t \to \infty$.

1. What is $\lim_{x \to 0} f(x)$ for $f(x) = \frac{x^3 - 1}{x^2 - x}$? Do the one-sided limits equal each other?

In [2]:
```
clear_vars()
#1
var('t')
func = sqrt(t)-t^2+cos(4*t)
```

In [4]:
```
clear_vars()
#2
x = var('x')
f = (x^3-1)/(x^2-x)
```

2. The Solve function

The solve command has many different applications - it's a good multipurpose tool for isolating variables. As you can find in the *Glossary of Commands*, there are two arguments needed for *solve()*: the expression you're given, and what you're solving for.

Solve the quadratic $x^2 + x - 5 = 0$ for x. Remember to use the double == when you're denoting an actual mathematical equality, as in an equation.

In [ ]:
```

```

If we have a *system of equations* (any linear algebra students in the establishment?!) we can solve for multiple variables, assuming we have enough information about those variables. Find x, y, and z in the following system:

$$x - y = 2$$
$$y - 3z = 1$$
$$x + y + z = 4$$

In [ ]:
```

```

One last thing about solving. In a situation where we *don't* have enough information to find what a variable is equal to, we can still use Sage to isolate the variable, rewriting it in terms of everything else. This can be especially useful when we want to find inverse equations.

In [12]:
```
# a simple example: what is the inverse of cos(x)?
solve( y==cos(x), x)
# arccos(y)
# Note that we can write this by hand as cos^-1(y), but Sage will not under
stand this notation.
```

Out[12]:
```
[x == arccos(y)]
```

Exercise questions

1. Solve for z in the equation $\frac{y}{z} + z^2 = x$.

   - Once you see your output, you'll be glad you didn't attempt to do this by hand!
   - You'll notice in two of the solutions, there is a little *i*. That means there is an imaginary number. Not a real number. For this course, you can disregard imaginary solutions.

1. Solve for x and y given that $9x + 2y = 1$ and $x - y^2 = 0$. Then plot the two curves together.

   - Wherever the curves intersect, there is a solution for the system of equations.

In [ ]:

```

```

That's it! That's all.

In [ ]:

```

```