Today: 18/11/25

More integration by parts

Ex1: $\int e^x \sin(x)\,dx = e^x\sin(x) - \int e^x\cos(x)\,dx$

$u = \sin(x)$ $v' = e^x$

$u' = \cos(x)$ $v = e^x$

$s = \cos(x)$ $t' = e^x$

$s' = -\sin(x)$ $t = e^x$

$$\int uv'\,dx = uv - \int u'v\,dx$$

$$= e^x\sin(x) - \left[\int e^x\cos(x) - \int(-\sin(x))\,e^x\,dx\right]$$

$$= e^x(\sin(x) - \cos(x)) - \int e^x\sin(x)\,dx$$

$$= 2\int e^x\sin(x)\,dx$$

$$= \frac{e^x(\sin(x) - \cos(x))}{2} + C$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

Ex2: $\int x^2\cosh(x)\,dx = x^2\sinh(x) - \int 2x\sinh(x)\,dx$

$u = x^2$ $v' = \cosh(x)$

$u' = 2x$ $v = \sinh(x)$

$s = 2x$ $t' = \sinh(x)$

$s' = 2$ $t = \cosh(x)$

$$= x^2\sinh(x) - \left[2x\cosh(x) - \int 2\cosh(x)\,dx\right]$$

$$= x^2\sinh(x) - 2x\cosh(x) + 2\cosh(x) + C$$

Ex3: $\int \frac{x\sin(x)}{1+\cos^2(x)}\,dx$

$u = x$ $v' = \frac{\sin(x)}{1+\cos^2(x)}$

$u' = 1$ $v = -\arctan(\cos(x)) + C$

$$\int \frac{\sin(x)}{1-\cos^2(x)}\,dx = \int \frac{1}{1+w^2}(-1)\,dx$$

$w = \cos(x)$ $= -\arctan(w) + C$

$dw = -\sin(x)\,dx$ $= -\arctan(\cos(x)) + C$

$(-1)\,dw = \sin(x)\,dx$

✗ not the right way

$$= -x\arctan(\cos(x)) - \int 1\cdot(-\arctan(\cos(x)))\,dx$$

$$= -x\arctan(\cos(x)) + \int \arctan(\cos(x))\,dx$$