Today: 10/11/25

Integration continued

Ex1: $\int_1^4 (x^2+x+1)\,dx$ → the area under the curve and x axis between $x=1$ to $x=4$

→ to integrate we take the antiderivative of the function

$= F(4) - F(1)$ where $F'(x) = x^2+x+1$

$$F(x) = \frac{x^{2+1}}{2+1} + \frac{x^{1+1}}{1+1} + \frac{x^{0+1}}{0+1}$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + x$$

$$= \left(\frac{(4)^3}{3} + \frac{(4)^2}{2} + 4\right) - \left(\frac{(1)^3}{3} + \frac{(1)^2}{2} + 1\right)$$

$$= \frac{64}{3} + \frac{16}{2} + 4 - \frac{1}{3} - \frac{1}{2} + 1$$

$$= \frac{63}{3} + \frac{15}{2} + 3$$

$$= 31.5$$

Ex 2: $\int_0^{\pi/2} [\sin(x)+\cos(x)]\,dx$

$\Rightarrow \int \cos(x)\,dx = \sin(x) + C$

$\Rightarrow \int \sin(x)\,dx = -\cos(x) + C$

$\frac{d}{dx}\sin(x) = \cos(x)$

$\frac{d}{dx}\cos(x) = -\sin(x)$

$$= \left[-\cos(x) + \sin(x) + C\right]\Big|_0^{\frac{\pi}{2}}$$

$$= \left[-\cos\left(\tfrac{\pi}{2}\right) + \sin\left(\tfrac{\pi}{2}\right) + \cancel{C}\right] - \left[-\cos(0) + \sin(0) + \cancel{C}\right]$$

$$= [0+1] - [-1+0] = 2$$

library of antiderivatives integrals
(i.e. generic antiderivatives)

1° (Power Rule) $\int x^n\,dx = \begin{cases} \frac{x^{n+1}}{n+1} & n \neq -1 \\ \ln(x) & n=-1 \end{cases} + C$

2° (Trig) $\int \cos(x)\,dx = \sin(x) + C$

$\int \sin(x)\,dx = -\cos(x) + C$

3° $\int e^x\,dx = e^x + C$

4° $\int \sec(x)\,dx = \ln(\sec(x) + \tan(x)) + C$

→ proof: $\frac{d}{dx}\ln(\sec(x)+\tan(x))$

$$= \frac{1}{\sec(x)+\tan(x)} \cdot \frac{d}{dx}(\sec(x)+\tan(x))$$

$$= \frac{\sec(x)\tan(x) + \sec^2(x)}{\sec(x)+\tan(x)}$$

$$= \frac{\sec(x)\cancel{(\tan(x)+\sec(x))}}{\cancel{\sec(x)+\tan(x)}}$$

$$= \sec(x)$$

5° $\int \tan(x)\,dx = \ln(\sec(x)) + C$

→ proof: $\frac{d}{dx}\ln(\sec(x))$

$$= \frac{1}{\sec(x)} \cdot \frac{d}{dx}(\sec(x))$$

$$= \frac{\cancel{\sec(x)}\tan(x)}{\cancel{\sec(x)}} = \tan(x)$$

6° $\int \ln(x)\,dx = x\ln(x) - x + C$

→ proof: $\frac{d}{dx}(x\ln(x) - x)$

$$= (x)'(\ln(x)) + (x)(\ln(x))' - 1$$

$$= \ln(x) + \cancel{x}\left(\frac{1}{\cancel{x}}\right) - 1$$

$$= \ln(x) + 1 - 1$$

$$= \ln(x)$$