Today: 07/11/25

Integration ✱ will not be on the test (Nov 14th

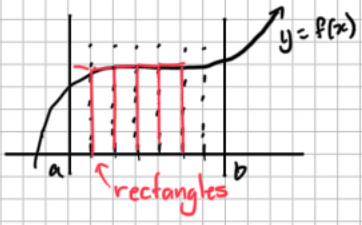
↳ how to compute areas, volumes and parabola's

$A = \Delta + \frac{1}{4}\Delta + \frac{1}{16}\Delta$

$= \Delta\left(1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots\right) = \frac{\Delta}{1 + \frac{1}{4}}$

$a = \Delta$

$r = \frac{1}{4}$



Area under $y = f(x)$ for $a \le x \le b$

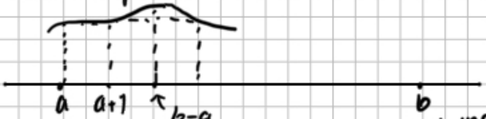
Divide up the area into rectangles, then take the limit of the sum of the areas of the rectangle

↳ = area under $y = f(x)$

$n \to \infty$

Right-hand Rule: divide up the interval $[a, b]$ into equal pieces, & the height of the n^{th} piece you get by evaluating $f(x)$ at the right hand endpoint of each point.

$\frac{b-a}{n}$ width of each piece



$\frac{b-a}{n}$

THIS # inc.

$$\underbrace{\frac{b-a}{n} \cdot f\left(a + 1 \cdot \frac{b-a}{n}\right)}_{\text{1st width}} + \underbrace{\frac{b-a}{n} \cdot f\left(a + 2 \cdot \frac{b-a}{n}\right)}_{\text{2nd width}} \dots$$

$$= \sum_{i=1}^{n} \frac{b-a}{n} \cdot f\left(a + i\,\frac{b-a}{n}\right)$$

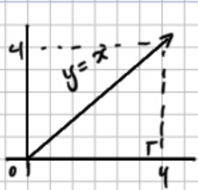
sum

area between $y = f(x)$ and the x-axis for

$$a \le x \le b = \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{\frac{b-a}{n} \cdot f\left(a + i\,\frac{b-a}{n}\right)}_{\text{area of the rectangle}}$$

$$= \int_a^b f(x)\,dx$$

Ex:



$A = \frac{b \cdot h}{2} = \frac{4 \cdot 4}{2} = 8$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4-0}{n} \cdot f\left(0 + i\,\frac{4-0}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n} \cdot f\left(i \cdot \frac{4}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{16}{n^2}\, i$$

$$= \lim_{n\to\infty} \frac{16}{n^2}\left(\sum_{i=1}^{n} i\right)$$

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

grouping

$\boxed{1} + \boxed{2} + \boxed{3} \dots + \boxed{(n-1)} + \boxed{n}$

$\boxed{n} + \boxed{(n-1)} + \boxed{(n-2)} + \dots + \boxed{2} + \boxed{1}$

$= (n+1) + (n+1) + (n+1) + \dots + (n+1) + (n+1)$

$= n(n+1)$

$$= \lim_{n\to\infty} \frac{16}{n^2} \cdot \frac{n(n-1)}{2}$$

$$= \lim_{n\to\infty} \frac{16n + 16}{2n}$$

$$= \lim_{n\to\infty} \left(8 + \frac{8}{n}\right) = 8$$

✱ Area under the x axis is considered negative

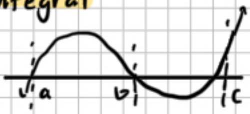
↳ $\int_a^b f(x)\,dx$ = weighted area between $y = f(x)$ & the x axis for $a \le x \le b$

↳ area above x axis is positive
area below x axis is negative

Properties of the definite integral

1° $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx \rightarrow$

$= \int_a^c f(x)\,dx$



2° $\int_a^b [f(x)+g(x)]\,dx \rightarrow \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

3° $\int_a^b c\,f(x)\,dx \rightarrow c\int_a^b f(x)\,dx$

← a to b = negative

4° $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

5° $\int_a^a f(x)\,dx = 0$

→ if $F(x) = \int_a^x f(t)\,dt$, then $F'(x) = f(x)$ version 2

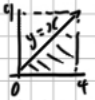
this is the fundamental theorem of calculus (II)

version 1 (I) of the fundamental theorem of calculus is:

if $F'(x) = f(x)$, then $\int_a^b f(x)\,dx = F(b) - F(a)$

anti derivative of $f(x)$

ex: $\int_0^4 x\,dx = \frac{x^2}{2}\Big|_0^4 = \frac{(4)^2}{2} - \frac{(0)^2}{2} = 8$



because $\frac{d}{dx}\left(\frac{x^2}{2}\right) = x$

★ example of the power rule

↳ Power Rule for Integrals

$$\int x^n\,dx = \begin{cases} \frac{x^{n-1}}{n+1} + C & n \neq -1 \\ \ln(x) + C & n = -1 \end{cases}$$

indefinite integral

generic antiderivative

Some functions do not have nice antiderivatives

e.g. $\frac{1}{\sqrt{2\pi}}\int e^{-x^2/2}$ → has no nice closed form

density of the standard normal distribution