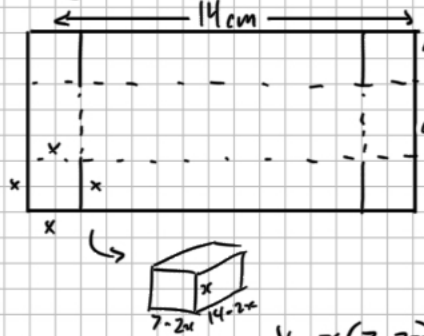
Today: 3/11/25

More optimization & related rates

Ex1: A rectangular piece of cardboard is to be made into a box by cutting into the corners and folding along the dotted lines.



what is the maximum possible volume of the box

$V = h \times w \times l$

$V = x(7-2x)(14-2x)$

$0 < x \leq 3.5$

$$V = x(7-2x)(14-2x)$$
$$= (7x-2x^2)(14-2x)$$
$$= 98x - 14x^2 - 28x^2 + 4x^3$$
$$= 4x^3 - 42x^2 + 98x$$

$$\frac{dV}{dx} = 12x^2 - 84x + 98$$
$$0 = 2(6x^2 - 42x + 49)$$
$$x = \frac{42 \pm \sqrt{(-42)^2 - 4(6)(49)}}{2(6)}$$
$$= \frac{42 \pm \sqrt{1764 - 1176}}{12}$$
$$= \frac{42 + \sqrt{588}}{12} = 5.52$$
$$= \frac{42 - \sqrt{588}}{12} = 1.48$$

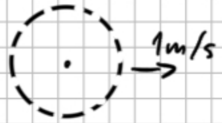
is in the interval

$V(0) = 0$

$V(3.5) = 0$

$V\left(\frac{42-\sqrt{588}}{12}\right) = 66.1L$

Ex2:



Drop a pebble into a still pod, creating a circular ripple that moves outward at 1m/s

1. How is the perimeter of the ripple changing after 2sec?
2. How is the area enclosed by the ripple changing after 2 seconds?

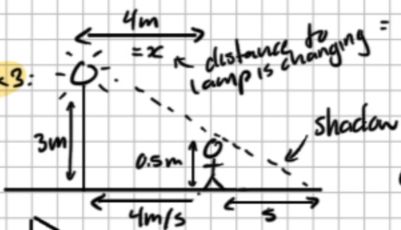
perimeter: $2\pi r$

$$\frac{dp}{dt} = \frac{d}{dt}(2\pi r) \qquad \frac{dr}{dt} = 1$$
$$= 2\pi \frac{dr}{dt}$$
$$= 2\pi 1 \frac{m}{s}$$

↳ Area $= \pi r^2$

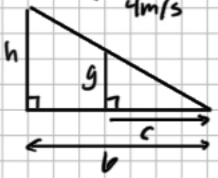
$$\frac{dA}{dt} = \frac{d}{dt}\pi r^2 \qquad r = 2$$
$$= \pi \cdot 2r \cdot \frac{dr}{dt}$$
$$= \pi \cdot 2r \cdot 1$$
$$= \pi \cdot 2 \cdot 2 \cdot 1$$
$$= 4\pi$$

Ex3:



1. How is the length of the shadow changing at this instant?

$Q: \left.\frac{ds}{dt}\right|_{x=4} = ?$



$\frac{h}{b} = \frac{g}{c}$

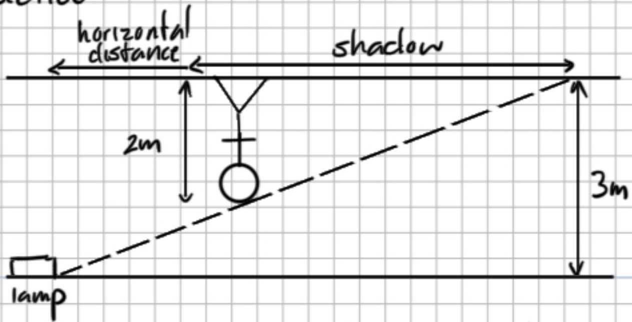
proportional

$$\frac{3}{0.5} = \frac{x+s}{s} \qquad x = 4$$
$$s \cdot 6 = x + s$$
$$x = 5s \quad \text{or} \quad s = \frac{x}{5}$$
$$\frac{ds}{dt} = \frac{d}{dt}\left(\frac{x}{5}\right) = \frac{1}{5}\frac{dx}{dt} = -\frac{4}{5}$$

Today: 4/11/25

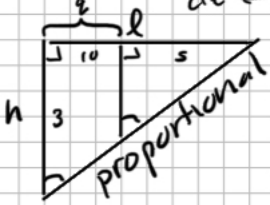
Practice

6.



how is Joe's shadow changing with time?

@10m, find $\frac{ds}{dt}$ ← shadow / ← time



$h = 3$

$l = 10 + s$ → $\frac{10+s}{3} = \frac{s}{2}$

$2(10+s) = 3s$

$20 + 2s = 3s$

$20 = s$

↖ at that instant but not the rate that it is changing

↳ $l = q + s$

$\frac{q+s}{3} = \frac{s}{2}$

$2(q+s) = 3s$

$2q = s$

$\frac{dq}{dt} = 10$

Solving for the derivative

↳ $\frac{d}{dt}(2q = s) = 2 \cdot \frac{dq}{dt} = \frac{ds}{dt}$

$2 \cdot 10 = \frac{ds}{dt}$

$20 \frac{m}{s} = \frac{ds}{dt}$

7.

$h = r$

$\frac{dv}{dt} = 0.1 \frac{m^3}{min}$

@ $h = 1m$

1. rate of change for the height of the cone

$V = \frac{1}{3}\pi r^2 h \rightarrow = \frac{\pi r^3}{3}$

differentiating with respect to time

$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{\pi r^3}{3}\right) \longrightarrow \frac{dV}{dt} = \frac{\pi}{3} \cdot \frac{d}{dt}(r^3)$

$= \frac{\pi}{\cancel{3}} \cdot \cancel{3} r^2 \cdot \frac{dr}{dt}$

$\frac{dv}{dt} \rightarrow 0.1 = \pi (1)^2 \cdot \left(\frac{dr}{dt}\right)$

↖ r

$\frac{dh}{dt} = \frac{dr}{dt} = \frac{0.1}{\pi}$ or $\frac{1}{10\pi}$ $\frac{m}{min}$