

Today: 17/10/25

## Practice

Ex1:  $y = (x-2)^{2/3}$  or  $\sqrt[3]{(x-2)^2}$

1) Domain: all real numbers  $\rightarrow x \in (-\infty, \infty)$

2) Intercepts: y int:  $(x=0) \rightarrow y = (0-2)^{2/3}$   
 $= [(-2)^{1/3}]^2$   
 $= 2^{2/3}$   
 x int:  $(y=0) \rightarrow 0 = (x-2)^{2/3}$   
 only if  $x=2$

3) Asymptotes:

$\rightarrow$  Vertex:  $y = (x-2)^{2/3}$  is continuous for all  $x$ , so there are no vertical asymptotes

$\rightarrow$  horizontal:  $\lim_{x \rightarrow -\infty} (x-2)^{2/3} \leftarrow -\infty - 2 = -\infty$   
 $\hookrightarrow [-\infty]^{1/3}$   
 $\hookrightarrow [-\infty]^2 = +\infty$   
 $\lim_{x \rightarrow \infty} (x-2)^{2/3} \leftarrow \infty - 2 = \infty$   
 $\hookrightarrow [\infty]^{1/3}$   
 $\hookrightarrow [\infty]^2 = +\infty$

4) Max/Min/inc./dec.

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} (x-2)^{2/3} \quad \leftarrow \frac{d}{dx} (x-2) \\ &= \frac{2}{3} (x-2)^{-1/3} \cdot (1) \\ &= \frac{2}{3} (x-2)^{-1/3} \\ &= \frac{2}{3(x-2)^{1/3}} \quad \text{find when } y=0 \end{aligned}$$

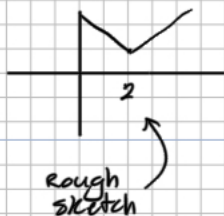
$\Leftrightarrow$  never = 0

$\hookrightarrow$  but is undefined at  $x=2$   
 when  $x < 2$ , then  $x-2 < 0$ , so  $\frac{2}{3(x-2)^{1/3}} < 0$ , so  $\frac{2}{3(x-2)^{1/3}} < 0$

when  $x > 2$ , then  $x-2 > 0$ , so  $\frac{2}{3(x-2)^{1/3}} > 0$ , so  $\frac{d}{dx} > 0$

| $x$             | $(-\infty, 2)$ | $2$ | $(2, \infty)$ |
|-----------------|----------------|-----|---------------|
| $\frac{dy}{dx}$ | -              | }   | +             |
| $y$             | $\downarrow$   | }   | $\uparrow$    |

$\leftarrow$  undefined



5) Inflection/CH/CD

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{d}{dx} \left( \frac{2}{3(x-2)^{1/3}} \right) \quad \leftarrow \frac{d}{dx} (x-2) \\ &= \frac{d}{dx} \left( \frac{2}{3} (x-2)^{-1/3} \right) \\ &= \frac{2}{3} \left( -\frac{1}{3} (x-2)^{-4/3} \right) (1) \\ &= -\frac{2}{9} (x-2)^{-4/3} \quad \text{or} \quad -\frac{2}{9(x-2)^{4/3}} \end{aligned}$$

$$y'' = \frac{-2}{9(x-2)^{4/3}}$$

$$x < 2 \Rightarrow (x-2)^{4/3} > 0$$

$$\Rightarrow y'' < 0 \quad \leftarrow \text{everything is negative}$$

$$x > 2 \Rightarrow (x-2)^{4/3} > 0$$

$$\Rightarrow y'' < 0 \quad \leftarrow \text{still less than 0}$$

| $x$   | $(-\infty, 2)$ | $2$ | $(2, \infty)$ |
|-------|----------------|-----|---------------|
| $y''$ | -              | }   | -             |
| $y$   | $\cap$         | }   | $\cap$        |

$\leftarrow$  undefined

$\rightarrow$  so no inflection points

