Today: 30/09/25

More derivatives

Ex 1 $\frac{d}{dx}\sin(3x)\cos(3x) = \left[\frac{d}{dx}\sin(3x)\right]\cos(3x) + \sin(3x)\left[\frac{d}{dx}\cos(3x)\right]$

$= \cos(3x)\cdot\left[\frac{d}{dx}3x\right]\cdot\cos(3x) + \sin(3x)\left[\frac{d}{dx}3x\right]\cdot(-\sin(3x))$

$= 3\cos^2(3x) - 3\sin^2(3x)$

$= 3(\cos^2(3x) - \sin^2(3x))$

$= 3\cos(2\cdot 3x)$

$= 3\cos(6x)$ ← Trig identities ↳ $\cos(2t) = \cos^2(t) - \sin^2(t)$

Ex 2 $\frac{d}{dx}\arctan(x^2+3x+1) = \frac{1}{1+(x^2+3x+1)^2}\cdot\left[\frac{d}{dx}(x^2+3x+1)\right]$

$= \frac{1}{1+(x^2+3x+1)^2}\cdot(2x+3)$

$= \frac{2x+3}{1+(x^2+3x+1)^2}$

$= \frac{2x+3}{1+x^4+3x^3+x^2+3x^3+9x^2+3x+x^2+3x+1}$

$= \frac{2x+3}{x^4+6x^3+11x^2+6x+2}$

Ex 3 $\frac{d}{dx}\left[e^{\cos(x)}\cdot\sin(e^x)\right]$

$= \left[\frac{d}{dx}\left[e^{\cos(x)}\right]\right]\cdot\sin(e^x) + e^{\cos(x)}\cdot\left[\frac{d}{dx}\sin(e^x)\right]$

$= e^{\cos(x)}\cdot(-\sin(x))\cdot\sin(e^x) + e^{\cos(x)}\cdot\cos(e^x)\cdot e^x$

$= e^{\cos(x)}\left(-\sin(x)\cdot\sin e^x + \cos(e^x)\cdot e^x\right)$

$= -e^{\cos(x)}\sin(x)\sin(e^x) + e^{\cos(x)+x}\cdot\cos(e^x)$ (or $\sin(e^x)$)

Ex 4 $\frac{d}{dx}\tan(\arctan(x))$

$= \sec^2(\arctan(x))\cdot\frac{d}{dx}\arctan(x)$

$= \left[1+\tan^2(\arctan(x))\right]\cdot\frac{1}{1+x^2}$

$= \left[1+x^2\right]\cdot\frac{1}{1+x^2} = 1$

→ $\frac{d}{dx}\tan(t) = \sec^2(t)$

→ $\frac{d}{dx}f^{-1}(f(x)) = \frac{d}{dx}x = 1$

$\frac{d}{dx}f(f'(x)) = \frac{d}{dx}x = 1$

$\frac{d}{dx}f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$