Today: 29/09/25

Practice - Computing derivatives

Ex: $\frac{d}{dx}\left(\frac{x^2-4}{x^2+x-2}\right) = \frac{(x^2-4)'(x^2+x-2)-(x^2+x-4)'(x^2-4)}{(x^2+x-2)^2}$

$= \frac{2x(x^2+x-2)-(2x+1)(x^2-4)}{(x^2+x-2)^2}$

$= \frac{\cancel{2x^3}+2x^2-4x-\cancel{2x^3}+8x-x^2+4}{(x^2+x-2)^2}$

$= \frac{x^2+4x+4}{(x^2+x-2)^2}$

$= \frac{(x+2)^2}{[(x+2)(x-1)]^2}$

$= \frac{\cancel{(x+2)^2}}{\cancel{(x+2)^2}(x-1)^2} = \frac{1}{(x-1)^2}$

another approach: (easier / faster)

$= \frac{d}{dx}\left(\frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)}\right) = \frac{d}{dx}\left(\frac{x-2}{x-1}\right)$

$= \frac{(x-2)'(x-1)-(x-1)'(x-2)}{(x-1)^2}$

$= \frac{(x-1)-(x-2)}{(x-1)^2} = \frac{1}{(x-1)^2}$

Ex2: $\frac{d}{dx}\cos(e^x \cdot x^2) = -\sin(w)\cdot\frac{d}{dx}(x^2e^x)$

$u=\cos(w)$

$w=x^2e^x$

$= -\sin(x^2e^x)((x^2)'e^x+(e^x)'x^2)$

$= -\sin(x^2e^x)(2xe^x+e^xx^2)$

$= -xe^x\sin(x^2e^x)(2+x)$

Ex3: $\frac{d}{dx}\, x^2\ln((x+1)^{-3})$

$= (x^2)'(\ln((x+1)^{-3})) + (x^2)(\ln((x+1)^{-3}))'$

$= 2x\cdot\ln((x+1)^{-3}) + x^2\cdot\left(\frac{1}{(x+1)^{-3}}\right)\cdot\left(\frac{d}{dx}((x+1)^{-3})\right)$

$= 2x\cdot\ln((x+1)^{-3}) + x^2\cdot(x+1)^3\cdot(-3)\cdot(x+1)^{-4}$

$= 2x\cdot\ln((x+1)^{-3}) - \frac{3x^2}{(x+1)}$

$\boxed{\ln(a^b)=b\ln(a)}$

$= -6x\cdot\ln(x+1) - \frac{3x^2}{(x+1)}$

another method:

$= \frac{d}{dx}\left[-3x^2\ln(x+1)\right]$

$= (-3x^2)'(\ln(x+1)) + (-3x^2)(\ln(x+1))'$

$= -6x\cdot\ln(x+1) - 3x^2\cdot\frac{1}{(x+1)}\cdot 1$

$= -6x\cdot\ln(x+1) - \frac{3x^2}{(x+1)}$