Today: 23/09/25

Epsilonics, rules for computing, limits and derivatives

1° Sum Rule

$$\lim_{x\to a}[f(x)+g(x)] = \left[\lim_{x\to a} f(x)\right] + \left[\lim_{x\to a} g(x)\right]$$

(provided they all exist)

proof: Assume that $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$

To show: $\lim_{x\to a}[f(x)+g(x)] = L+M$

$\lim_{x\to a} f(x) = L$
means for all $\varepsilon_f > 0$, there is a $\delta_f > 0$, s.t. if $|x-a| < \delta_f$, then $|f(x)-L| < \varepsilon_f$

$\lim_{x\to a} g(x) = M$
means for all $\varepsilon_g > 0$, there is a $\delta_g > 0$, s.t. if $|x-a| < \delta_g$, then $|g(x)-L| < \varepsilon_g$

$\lim_{x\to a}[f(x)+g(x)] = L+M$
means for all $\varepsilon > 0$, there is a $\delta > 0$, s.t. if $|x-a| < \delta$, then $|(f(x)+g(x)) - (L+M)| < \varepsilon$

Suppose that $\varepsilon > 0$, we reverse engineer the necassary $\delta > 0$

$$|(f(x)+g(x)) - (L+M)| < \varepsilon$$
$$\iff |(f(x)-L) + (g(x)-M)| < \varepsilon$$
$$\iff |f(x)-L| + |g(x)-M| < \varepsilon$$

$\leq$ to the statement above

$\varepsilon_f + \varepsilon_g = \varepsilon$

let $\varepsilon_f = \frac{\varepsilon}{2}$ and $\varepsilon_g = \frac{\varepsilon}{2}$

find the δ_f and δ_g for these

so long as $|x-a| < \min(\delta_f, \delta_g)$

Note: if $f(x)$ has the limit $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ exist the limit is the derivative of f at x

Rules for derivatives

- **Power Rule**

$$\frac{d}{dx} x^n = n x^{n-1}$$

- **Product Rule**

$$\frac{d}{dx}(f(x)g(x)) = \left[\frac{d}{dx} f(x)\right] g(x) + f(x)\left[\frac{d}{dx} g(x)\right]$$

- **Quotient Rule**

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

- **Sum Rule**

$$\frac{d}{dx}(f(x)+g(x)) = \left(\frac{d}{dx} f(x)\right) + \left(\frac{d}{dx} g(x)\right)$$

- **Constant Rule**

$$\frac{d}{dx}(c f(x)) = c\left(\frac{d}{dx} f(x)\right)$$

- **Chain Rule**

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

Ex: $\frac{d}{dx} \ln(\cos(x)) = \frac{1}{\cos(x)} \cdot (-\sin(x))$

$\boxed{\frac{d}{dx} \ln(x) = \frac{1}{x}}$ $\boxed{\frac{d}{dx} \cos(x) = -\sin(x)}$