

Today: 22/09/25

Limits

$$\begin{aligned} \text{Ex 1: } \lim_{x \rightarrow 0} \frac{e^x - 1}{x} &= 1 \\ &= \lim_{x \rightarrow 0} \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots \right) \cdot \frac{1}{x} \\ &= \lim_{x \rightarrow 0} \left(\frac{1}{x} + 1 + \frac{x}{2} + \frac{x^2}{6} + \dots \right) \leftarrow \text{factor in 0} \\ &= \lim_{x \rightarrow 0} \left(1 + \frac{0}{2} + \frac{0}{6} + \dots \right) \\ &= 1 \end{aligned}$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots + \frac{x^n}{n!} + \dots$$

$n! = n(n-1)(n-2)\dots \leftarrow \text{"n factorial"}$

$$\text{Ex 2: } \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 5x + 6} = \lim_{x \rightarrow 2} \frac{(x-2)(x+1)}{(x-2)(x-3)} = \lim_{x \rightarrow 2} \frac{x+1}{x-3} = \frac{2+1}{2-3} = -3$$

Ex 3: long division trick: $3x^4 + 2x^3 + x^2 - 1$

$$\begin{array}{r} 3x^2 - 4x + 12 \\ x^2 + 2x - 1 \overline{) 3x^4 + 2x^3 + x^2 - 1} \\ \underline{-3x^4 + 6x^3 - 3x^2} \\ 0 - 4x^3 + 4x^2 + 0x \\ \underline{-4x^3 + 8x^2 + 4x} \\ 0 + 12x^2 - 4x - 1 \\ \underline{-12x^2 + 24x - 12} \\ 0 - 28x + 11 \end{array}$$

remainder $\downarrow$

$$(-3x^2 - 4x + 12)(x^2 + 2x - 1) + (-28x + 11)$$

Ex 4: $i^2 = -1$

$$\begin{aligned} e^{ix} &= \cos(x) + i\sin(x) \\ &= \cos(nx) + i\sin(nx) \\ (e^{ix})^n &= (\cos(x) + i\sin(x))^n \end{aligned}$$

$$\begin{aligned} e^{ix} &= 1 + ix - \frac{x^2}{2} - \frac{ix^3}{6} + \frac{x^4}{24} + \frac{ix^5}{120} \dots \\ &= \left(1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots \right) + i \left(x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \dots \right) \\ &= \cos(x) + i\sin(x) \end{aligned}$$

Ex 5: Practice similar to the test

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{x^2 - 1}{x^3 - x^2 + x - 1} &= \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{[x^3 - x^2] + [x - 1]} = \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{x^2(x-1) + (x-1)} \\ &= \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{(x-1)(x^2 + 1)} \rightarrow \lim_{x \rightarrow 1} \frac{(x+1)}{(x^2 + 1)} \\ &= \frac{1+1}{1^2 + 1} = 1 \end{aligned}$$

$\leftarrow$ long division way to factor

$$\begin{aligned} \text{Ex 6: } \lim_{x \rightarrow 0} \frac{\tan(2x)}{\sin(x)} &= \lim_{x \rightarrow 0} \frac{\sin(2x)}{\cos(2x)} \cdot \frac{1}{\sin(x)} \\ &= \lim_{x \rightarrow 0} \frac{2\sin(x)\cos(x)}{\sin(x)\cos(2x)} \\ &= \lim_{x \rightarrow 0} \frac{2\cos(x)}{\cos(2x)} \\ &= \frac{2\cos(0)}{\cos(2 \cdot 0)} = \frac{2}{1} = 2 \end{aligned}$$

extra practice
 $\downarrow$
section 1
2.3 text books