# Epsilonics Practice

Sept 1

**ex.** $\lim_{x\to 0}(-x^2+6x-5)=-5$

for each $\varepsilon>0$ there is a $\delta>0$ such that if $|x-0|<\delta$ then $|(-x^2+6x-5)-(-5)|<\varepsilon$

iff $|-x^2+6x|<\varepsilon$

iff $|x(6-x)|<\varepsilon$

iff $|x|\cdot|6-x|<\varepsilon$

iff $|x-0|<\frac{\varepsilon}{|6-x|}$ we shall accept only $\delta<\frac{1}{4}$

if $|x-0|<\delta<\frac{1}{4}$, then $-\frac{1}{4}<x<\frac{1}{4}$

so $5.75<6-x<6.25$

so $\frac{1}{5.75}>\frac{1}{6-x}>\frac{1}{6.25}$

so $\frac{\varepsilon}{|6-x|}>\frac{\varepsilon}{6.25}$

so let $\delta=\min(\frac{1}{4},\frac{\varepsilon}{6.25})$

so if $|x-0|<\delta$ we have $|x-0|<\frac{1}{4}$ and $|x-0|<\frac{\varepsilon}{6.25}<\frac{\varepsilon}{|6-x|}$

and therefore $|(-x^2+6x-5)-(-5)|<\varepsilon$, so the limit is correct

**ex.** $\lim_{x\to 2}(2x^2-3x+5)=7$

for each $\varepsilon>0$ there is a $\delta>0$ such that if $|x-2|<\delta$ the $|(2x^2-3x+5)-7|<\varepsilon$

iff $|2x^2-3x-2|<\varepsilon$

iff $2|x^2-\frac{3}{2}x-1|<\varepsilon$

iff $|(x-2)(x+\frac{1}{2})|<\frac{\varepsilon}{2}$

iff $|x-2|<\frac{\varepsilon}{2|x+\frac{1}{2}|}$ only accept $\delta<\frac{1}{10}$

if $|x-2|<\delta<\frac{1}{10}$, then $1.9<x<2.1$

thus $2.4<|x+\frac{1}{2}|<2.6$

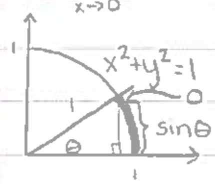
so $\frac{1}{2.4\cdot 2}>\frac{1}{2|x+\frac{1}{2}|}>\frac{1}{2.6\cdot 2}$

so $\delta=\min(0.1,\frac{\varepsilon}{5.2})$

so if $|x-2|<\delta$ we have $|x-2|<0.1$ and $|x-2|<\frac{\varepsilon}{5.2}<\frac{\varepsilon}{2|x+\frac{1}{2}|}$

and therefore $|(2x^2-3x+5)-7|<\varepsilon$, so the limit is correct

**ex.** $\lim_{x\to 0}\sin(x)=0$




$0\le\sin\theta\le\theta$

$\downarrow$

$0$