Today: 15/04/23

Practice day

quadratic epsilonics

↳ $\lim_{x \to 0} (-x^2 + 6x - 5) = -5$

verify this using the ε-δ definition of limits...

$a = 0 \quad L = -5$

for each $\varepsilon > 0$ there is a $\delta > 0$, such that if $|x - 0| < \delta$ then, $|(-x^2 + 6x - 5) - (-5)| < \varepsilon$

$$|(-x^2+6x-5)-(-5)| < \varepsilon$$
$$\Leftrightarrow |-x^2+6x| < \varepsilon$$
$$\Leftrightarrow |x(6-x)| < \varepsilon$$
$$\Leftrightarrow |x| \cdot |6-x| < \varepsilon$$
$$\Leftrightarrow |x-0| < \frac{\varepsilon}{|6-x|}$$

We will only accept $\delta < \frac{1}{4}$

If $|x - 0| < \delta < \frac{1}{4}$

$$-\frac{1}{4} < x < \frac{1}{4}$$

$$5.75 < 6 - x < 6.25$$

so $5.75 < |6-x| < 6.25$

$$\boxed{\frac{1}{6.25} < \frac{1}{|6-x|}} < \frac{1}{5.75}$$

(a, b)

let $\delta = \min\left(\frac{1}{4}, \frac{\varepsilon}{6.25}\right)$

taking the min ensures that δ is small enough to meet both conditions $\delta \le a$ and $\delta \le b$

$$\frac{\varepsilon}{|6-x|} > \frac{\varepsilon}{6.25}$$

$$|x - 0| < \delta$$
$$|x - 0| < \frac{1}{4}$$
$$|x - 0| < \frac{\varepsilon}{6.25} < \frac{\varepsilon}{6-x}$$

$\lim_{x \to 2} (2x^2 - 3x + 5) = 7$

verify for each $\varepsilon > 0$ there is a $\delta > 0$, so if $|x - 2| < \delta$, then $|(2x^2 - 3x + 5) - 7| < \varepsilon$

$$|(2x^2-3x+5)-7| < \varepsilon$$
$$\Leftrightarrow |2x^2-3x-2| < \varepsilon$$
$$\Leftrightarrow 2|x^2-\tfrac{3}{2}x-1| < \varepsilon$$
$$\Leftrightarrow |(x-2)(x+\tfrac{1}{2})| < \frac{\varepsilon}{2}$$
$$\Leftrightarrow |x-2| < \frac{\varepsilon}{2|x+\frac{1}{2}|}$$

we will only accept that $\delta < \frac{1}{10}$

so if $|x - 2| < \delta < \frac{1}{10}$ we have..

$$-\frac{1}{10} < x - 2 < \frac{1}{10}$$
$$1.9 < x < 2.1$$

thus...

$$2.4 < x + \tfrac{1}{2} < 2.6$$

$$\boxed{\frac{1}{2 \times 2.6} < \frac{1}{2|x+\frac{1}{2}|}} < \frac{1}{2.4}$$

Don't care what's above

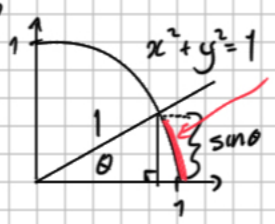
, so... $\delta = \min\left(0.1, \frac{\varepsilon}{5.2}\right)$

$|x - 2| < \delta$ it is also $< \frac{\varepsilon}{|x+\frac{1}{2}|}$

& thus $|(2x^2 - 3x + 5) - 7| < \varepsilon$

therefore the limit works

$\lim_{x \to 0} \sin(x) = 0$



arc length $= \theta$

$0 \le \sin(\theta) < \theta$

already at 0

heading to zero