More limits and epsilon's

Monday, September 15, 2025 7:53 AM

Quadratic Epsilonics Practice

① $\lim_{x\to 0}(-x^2-6x+5) = -5$ verify this using the ε-δ definition

→ may show up on assignments, not test/exams

for each $\varepsilon>0$, there is a $\delta>0$, such that if $|x-0|<\delta$ then $|(-x^2+6x-5)-(-5)|<\varepsilon$

iff $|-x^2+6x|<\varepsilon$
iff $|x(6-x)|<\varepsilon$
iff $|x|\cdot|6-x|<\varepsilon$
iff $|x-0|<\dfrac{\varepsilon}{|6-x|}$ ← can't work bc 0 in denominator
↳ δ can't depend on x if it needs to control x

→ so, we will accept only $\delta<\frac{1}{4}=0.2$

If $|x-0|<\delta<\frac{1}{4}$, then $-\frac{1}{4}<x<\frac{1}{4}$, so

$\underset{6-.25}{5.75} < \underset{\overset{\shortparallel}{|6-x|}}{6-x} < \underset{6+.25}{6.25}$ so

$\dfrac{1}{5.75} > \dfrac{1}{|6-x|} > \dfrac{1}{6.25}$ so

$\dfrac{\varepsilon}{|6-x|} > \dfrac{\varepsilon}{6.25}$ → if $\delta<.25$, whatever is smaller than $\frac{\varepsilon}{6.25}$ is also smaller than $\frac{\varepsilon}{|6-x|}$

→ so let $\delta=\min\left(\frac{1}{4}, \frac{\varepsilon}{6.25}\right)$

so if $|x-0|<\delta$ we have
$|x-0|<\frac{1}{4}$ and $|x-0|<\dfrac{\varepsilon}{6.25}<\dfrac{\varepsilon}{|6-x|}$

therefore, $|(-x^2+6x-5)-(-5)|<\varepsilon$ so the limit is correct.

② $\lim_{x\to 2}(2x^2-3x+5)=7$ verify!

For each $\varepsilon>0$, there is a $\delta>0$ s.t. if $|x-2|<\delta$, then $|(2x^2-3x+5)-7|<\varepsilon$

$|(2x^2-3x+5)-7|<\varepsilon$
iff $|2x^2-3x-2|<\varepsilon$ → $(x-2)(x+\frac{1}{2}) = x^2-\frac{3}{2}x-1$
iff $2|x^2-\frac{3}{2}x-1|<\varepsilon$
iff $|(x-2)(x+\frac{1}{2})|<\frac{\varepsilon}{2}$
iff $|x-2|<\dfrac{\varepsilon}{2|x+\frac{1}{2}|}$ → We'll ensure that $\delta<\frac{1}{10}=0.1$

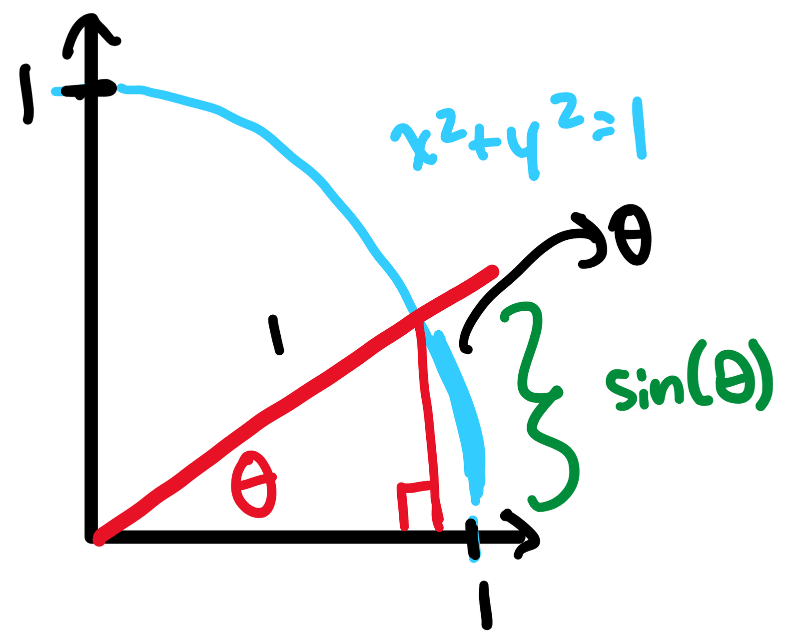
so if $|x-0|<\delta<\frac{1}{10}$ we have $1.9<x-2<2.1$ → thus $2.4<\underset{\overset{\shortparallel}{|x+\frac{1}{2}|}}{x+\frac{1}{2}}<2.6$

so $\dfrac{1}{2.4}>\dfrac{1}{|x+\frac{1}{2}|}>\dfrac{1}{2.6}$

so $\delta=\min\left(0.1, \dfrac{\varepsilon}{5.2}\right)$ ↳ (2.6×2)

so if $|x-2|<\delta$, it is also $<\dfrac{\varepsilon}{2|x+\frac{1}{2}|}$ and thus $|(2x^2-3x+5)-7|<\varepsilon$, and so the limit works

③ $\lim_{\theta\to 0}\sin(x)=0$



$0\le\sin(\theta)<\theta$

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