Limits and Epsilonics Sept

$\lim_{x\to a} f(x) = L$ means as x approaches a, f(x) gets close to L

definition: $\lim_{x\to a} f(x) = L$ means for all $\varepsilon > 0$ there is $\delta > 0$ such that if $|x-a| < \delta$ then $|f(x) - L| < \varepsilon$

ex. $f(x) = 5x+1$ $a = 12$ $L = 61$ $\varepsilon = \frac{1}{2}$

What does δ have to be to ensure that $|(5x+1)-61| < \frac{1}{2}$ whenever $|x-12| < \delta$

$|(5x+1) - 61| < \frac{1}{2}$

$\Leftrightarrow -\frac{1}{2} < 5x+1-61 < \frac{1}{2}$

$\Leftrightarrow -\frac{1}{2} < 5x-60 < \frac{1}{2}$

$\Leftrightarrow -\frac{1}{2} < 5(x-12) < \frac{1}{2}$

$\Leftrightarrow -\frac{1}{2}\cdot\frac{1}{5} < x-12 < \frac{1}{2}\cdot\frac{1}{5}$

$\Leftrightarrow -\frac{1}{10} < x-12 < \frac{1}{10}$

$\Leftrightarrow |x-12| < \frac{1}{10}$ any $\delta \le \frac{1}{10}$ will work

assume a generic ε

$|(5x+1) - 61| < \varepsilon$

$\Leftrightarrow -\varepsilon < 5x+1-61 < \varepsilon$ same steps

$\Leftrightarrow -\frac{\varepsilon}{5} < x-12 < \frac{\varepsilon}{5}$ as before

ex. $g(x) = 2x+9$ $a = 70$ $L = 149$

$\lim_{x\to 70} (2x+9) = 149$

is for any $\varepsilon > 0$, there is a $\delta > 0$ such that if $|x-70| < \delta$ then $|(2x+9) - 149| < \varepsilon$

$|(2x+9) - 149| < \varepsilon$

$\Leftrightarrow |2x - 140| < \varepsilon$

$\Leftrightarrow |2(x-70)| < \varepsilon$

$\Leftrightarrow |2|\cdot|x-70| < \varepsilon$ don't do

$\Leftrightarrow 2|x-70| < \varepsilon$ unless negative

$\Leftrightarrow |x-70| < \frac{\varepsilon}{2}$ any $\delta \le \frac{\varepsilon}{2}$ works

ex. $h(x) = -3x + \frac{1}{2}$ $a = 2$ $L = -\frac{11}{2}$

$\lim_{x\to 2} |(-3x+\frac{1}{2}) - (-\frac{11}{2})|$ for any $\varepsilon > 0$ there is a $\delta > 0$ s.t. if $|x-2| < \delta$ then $|(-3x+\frac{1}{2}) - (-\frac{11}{2})| < \varepsilon$

$|(-3x+\frac{1}{2}) - (-\frac{11}{2})| < \varepsilon$

iff $|-3x + \frac{1}{2} + \frac{11}{2}| < \varepsilon$

iff $|-3x+6| < \varepsilon$

iff $|-3(x-2)| < \varepsilon$

iff $3|x-2| < \varepsilon$

iff $|x-2| < \frac{\varepsilon}{3}$ any $\delta < \frac{\varepsilon}{3}$ works $\because$ every step is reversable ex. $h(x)=\sqrt{|x|}$ $\qquad \lim_{x\to 0}\sqrt{|x|}=0$

for every $\varepsilon>0$, there is a $\delta>0$ such that if $|x-0|<\delta$ then $|\sqrt{|x|}-0|<\varepsilon$

$|\sqrt{|x|}-0|<\varepsilon$

iff $|\sqrt{|x|}|<\varepsilon$

iff $\sqrt{|x|}<\varepsilon$

iff $(\sqrt{|x|})^2<\varepsilon^2$

$\quad |x| \qquad$ so $\delta\le\varepsilon^2$

ex. $p(x)=x^2$ $\qquad a=9 \qquad L=81$

for every $\varepsilon>0$, there is a $\delta>0$ such that if $|x-9|<\delta$ then $|x^2-81|<\varepsilon$

$|x^2-81|<\varepsilon$

iff $|(x-9)(x+9)|<\varepsilon$ $\quad$ we will accept no $\varepsilon>1$

$\quad |(x-9)(x+9)|\le 1$

$\quad |x^2-81|\le 1$

iff $-1\le x^2-81\le 1$

iff $80\le x^2\le 82$ $\qquad \because x>0$ near $a=9$

iff $\sqrt{80}\le x\le\sqrt{82}$

iff $9+\sqrt{80}\le x+9\le 9+\sqrt{82}$

iff $\frac{1}{9+\sqrt{82}}\le\frac{1}{x+9}\le\frac{1}{9+\sqrt{80}}$

iff $\frac{1}{9+\sqrt{82}}\le\frac{1}{|x+9|}\le\frac{1}{9+\sqrt{80}}$

we will accept no $\delta>1$

$|x-9|<\delta\le 1$

iff $-1\le x-9\le 1$

iff $8\le x\le 10$ $\qquad |x-9|<\frac{1}{19}\varepsilon\le\frac{\varepsilon}{|x+9|}$

iff $17\le x+9\le 19$

iff $17\le |x+9|\le 19$

iff $\frac{1}{19}\le\frac{1}{|x+9|}\le\frac{1}{17}$ $\qquad \delta=\min(1,\frac{1}{19})$

FIVE STAR ★★★★★