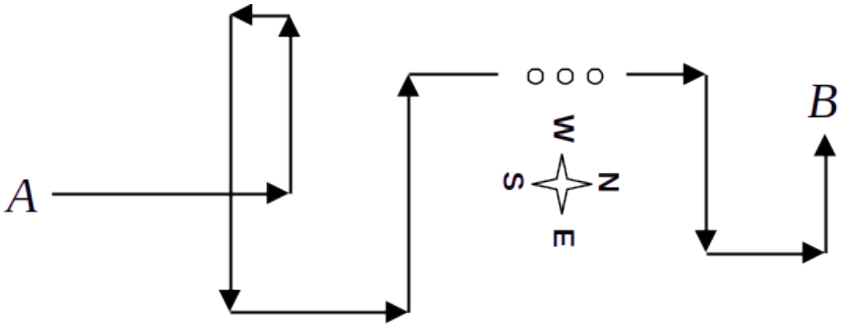
Mathematics 1110H (Section A) – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Fall 2024

Solutions to Quiz #1
Walk Away?

Nemo Sum walks from point A to point B and then back again, taking the exact same path in the opposite direction. Nemo always walks in one of the cardinal directions, *i.e.* north, south, east, or west, but not always at the same speed. For reasons known only to higher powers, Nemo always walks at 4 km/h when going North or South; going East, Nemo always walks at 3 km/h; and going West, Nemo always walks at 6 km/h.



Pieces of a possible path.

Nemo's walk, from the start at point A to the return at point A, lasts exactly 6 hours.

1. How long is the path taken by Nemo from point A to point$B$? *[3]*

NOTE. The question is asking for the length of the path traversed by Nemo, not the distance from A to B.

SOLUTION. Going North or South a kilometer takes 1/4 of an hour, going East a kilometer takes 1/3 of an hour, and going West a kilometer takes 1/6 of an hour. Hence to go and return over the same kilometer, whether North-South or East-West, takes $\frac{1}{2} = \frac{1}{4}+\frac{1}{4} = \frac{1}{3}+\frac{1}{6}$ of an hour. This means that each half hour of the total walk corresponds to walking 2 km, counting the outward and return legs together. Hence in 6 hours Nemo walked a total of $6 \times 2 \times 2 = 24$ km. Since this includes traversing the path twice, from point A to point B and then back again, the path is $24/2 = 12$ km long. □

2. How long after setting out from point A might Nemo have reached point B? *[2]*

Hint: The answer to this question should be a range of times, rather than a single time.

SOLUTION. If the 12 km of path from point A to pont B were all East, ir would take Nemo $12 \times \frac{1}{3} = 4$ hours to get to point B after setting out from point A; if the 12 km of path from point A to pont B were all West, ir would take Nemo $12 \times \frac{1}{6} = 2$ hours to get to point B after setting out from point A; if the 12 km of path from point A to pont B

were a mix of East, West, North, and/or South it would take somewhere between 2 and 4 hours for Nemo to get to point B after setting out from point A. (Why?) Thus Nemo would have reached point B between 2 and 4 hours after setting out from point B. ■

NOTE. This quiz is a modified version of the problems presented in Knot I of Lewis Carroll's *A Tangled Tale*, which you can find in a number of places online, including `http://euclid.trentu.ca/math/sb/carroll/tangled/title.html` . Lewis Carroll was the pen name of Charles Lutwidge Dodgson (1832-1898), best known nowadays as the author of *Alice's Adventures in Wonderland* and its sequel, *Through the Looking-Glass*. He juggled several careers as a mathematician, writer, poet, and photographer.