Mathematics 1110H (Section A) – Calculus I: Limits, Derivatives, and Integrals

TRENT UNIVERSITY, Fall 2024

Assignment #3
Differential Equations
*Due on Friday, 18 October.**

If you haven't already seen it, look up SageMath's `desolve` command, which is used to solve differential equations.

1. Consider the differential equation $x\dfrac{dy}{dx} = y$.

   a. Use SageMath to find a general solution to this differential equation. *[1]*

   b. Use SageMath to find a solution to this differential equation satisfying the initial condition $y = 1$ when $x = 1$. *[1]*

   c. Verify – by hand! – that the solution you obtained in part **b** satisfies the given differential equation with the given initial conditions. *[0.5]*

2. Consider the differential equation $\dfrac{dy}{dx} = xy$.

   a. Use SageMath to find a general solution to this differential equation. *[1]*

   b. Use SageMath to find a solution to this differential equation satisfying the initial condition $y = 1$ when $x = 0$. *[1]*

   c. Verify – by hand! – that the solution you obtained in part **b** satisfies the given differential equation with the given initial conditions. *[0.5]*

3. Consider the differential equation $\dfrac{dy}{dx} = y^2 + 1$.

   a. Use SageMath to help find a general solution to this differential equation. *[1]*

   b. Use SageMath to help find a solution to this differential equation satisfying the initial condition $y = 0$ when $x = 0$. *[1]*

   c. Verify – by hand! – that the solution you obtained in part **b** satisfies the given differential equation with the given initial conditions. *[0.5]*

4. Use SageMath to help find all the general solutions to the differential equation

$$\left(\frac{dy}{dx}\right)^2 + (x + y)\frac{dy}{dx} + xy = 0.$$

Explain why the solutions you found work. *[2.5]*

Hint: A little preliminary algebra can make this much easier.