Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
Section A, Trent University, Fall 2024

Final Examination

11:00-14:00 in the Gym on Tuesday, 10 December.

Instructions: Do both of parts **A** and **B**, and, if you wish, part **C**. Please show all your work, justify all your answers, and simplify these where you reasonably can. When you are asked to do k of n questions, only the first k that are not crossed out will be marked. *If you have a question, or are in doubt about something,* **ask!**

Aids: Any calculator, as long as it can't communicate with other devices; all sides of one letter- or A4-size sheet, with whatever you want written on it; your own brain.

Part A. Do all four (4) of **1**–**4**.

1. Compute $\frac{dy}{dx}$ as best you can in any four (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. $y = \arctan\left(x^3\right)$ **b.** $y = x^2 e^{-x}$ **c.** $y = \ln\left(\sec(x) + \tan(x)\right)$

d. $y = \sec^3\left(\arctan(x)\right)$ **e.** $y = \frac{3+x^2}{4+x^2}$ **f.** $y = \left(\sin(x) + \cos(x)\right)^2$

2. Evaluate any four (4) of the integrals **a**–**f**. *[20 = 4 × 5 each]*

a. $\displaystyle\int \frac{x}{\sqrt{x^2+1}}\,dx$ **b.** $\displaystyle\int_0^1 xe^{-x}\,dx$ **c.** $\displaystyle\int 2\ln(x)\,dx$

d. $\displaystyle\int_{-1}^1 (x+3)^3\,dx$ **e.** $\displaystyle\int \frac{x+1}{x^2-1}\,dx$ **f.** $\displaystyle\int_0^{\pi/2} \frac{\cos(x)}{1+\sin^2(x)}\,dx$

3. Do any four (4) of **a**–**g**. *[20 = 4 × 5 each]*

a. Compute $\lim_{x\to\infty} x^2 e^{-x}$.

b. Use the Right-Hand Rule to compute $\displaystyle\int_0^2 (x+1)\,dx$.

c. Use the ε–δ definition of limits to verify that $\lim_{x\to 2}(2x-3) = 1$.

d. Find the area of the region between $y = x^{1/3}$ and $y = x^3$, where $0 \le x \le 1$.

e. Determine whether $f(x) = \begin{cases} \frac{x^2}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases}$ is continuous at $x = 0$ or not.

f. Find the volume of the solid obtained by revolving the region between the line $y = 1$ and the line $y = x$, for $0 \le x \le 1$, about the y-axis.

g. Use the limit definition of the derivative to compute $g'(x)$ for $g(x) = x^2 + x$.

4. Find the domain, intercepts, vertical and horizontal asymptotes, intervals of increase and decrease, maximum and minimum points, intervals of concavity, and inflection points of $f(x) = x^2 e^{-x}$, and sketch its graph based on this information. *[12]*

There is more on page 2!

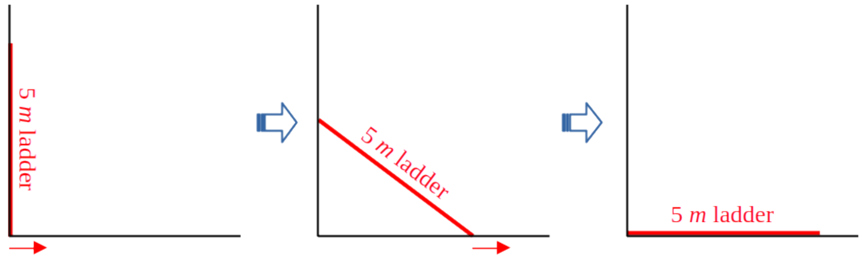
Part B. Do any two (2) of **5–7**. *[28 = 2 × 14 each]* *. . . and here is "more"!*

5. The region below $y = x^2$ and above $y = 0$, where $0 \leq x \leq 2$, is revolved about the line $x = -1$, making a solid of revolution.

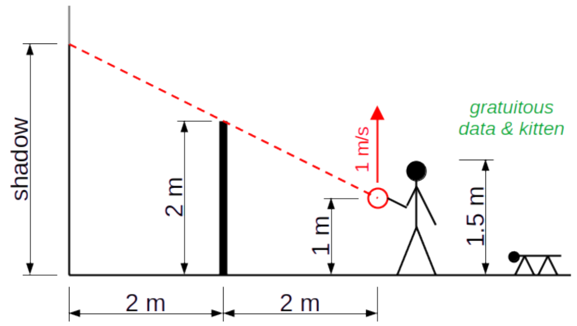
a. Sketch the region. *[1]* **b.** Find the area of the region. *[3]*

c. Sketch the solid. *[1]* **d.** Find the volume of the solid. *[9]*

6. A 5 m long ladder is flush up against a vertical wall at first. Its bottom then slides on the horizontal floor away from the wall, the top and bottom of the ladder maintaining contact with the wall and floor, respectively, until the ladder rests on the floor. What is the maximum area of the triangle made by the wall, floor, and ladder during this process? *[14]*



7. A vertical post 2 m tall stands on level ground 2 m from a vertical wall. Stick Person stands with a lantern that is 2 m from the post. Stick raises the lantern vertically at 1 m/s. How is the length of the shadow, as cast by the post onto the wall by the lantern's light, changing at the instant that the lantern is 1 m above the ground? *[14]*



[Total = 100]

Part C. *Bonus points!* Do one or both of $\mathbf{2^3}$ and $\mathbf{3^2}$.

$\mathbf{2^3}$**.** Write an original haiku touching on calculus or mathematics in general. *[1]*

What is a haiku?

seventeen in three:
five and seven and five of
syllables in lines

$\mathbf{3^2}$**.** Verify that $\ln(\sec(x) - \tan(x)) = -\ln(\sec(x) + \tan(x))$. *[1]*

APOLOGIES FOR ALL THE GLITCHES THIS TERM. HAVE A GOOD BREAK!