Mathematics 1110H (Section B) – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Fall 2023

Solutions to Quiz #5
More Derivatives

REMINDER. While you are allowed to work together and look things up when doing the quizzes and assignments, your submission should be written up entirely by yourself, giving credit to any collaborators or sources that you ended up actually using. *Please show all your steps and simplify your answers as far as practical.*

Using the practical rules for computing derivatives, find $\dfrac{dy}{dx}$ in each of the following questions.

1. $y = \dfrac{x^2-1}{x^2+1}$ *[1]*

SOLUTION. Quotient Rule and Power Rule:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2-1}{x^2+1}\right) = \frac{\left[\frac{d}{dx}\left(x^2-1\right)\right]\left(x^2+1\right) - \left(x^2-1\right)\left[\frac{d}{dx}\left(x^2+1\right)\right]}{\left(x^2+1\right)^2}$$
$$= \frac{[2x]\left(x^2+1\right) - \left(x^2-1\right)[2x]}{\left(x^2+1\right)^2} = \frac{2x^3+2x-2x^3+2x}{\left(x^2+1\right)^2} = \frac{4x}{\left(x^2+1\right)^2} \quad \square$$

2. $y = \sin\left((\ln(x))^2\right)$ *[1]*

SOLUTION. Chain Rule and Power Rule:

$$\frac{dy}{dx} = \frac{d}{dx}\sin\left((\ln(x))^2\right) = \cos\left((\ln(x))^2\right)\cdot\frac{d}{dx}(\ln(x))^2 = \cos\left((\ln(x))^2\right)\cdot 2\ln(x)\cdot\frac{d}{dx}\ln(x)$$
$$= \cos\left((\ln(x))^2\right)\cdot 2\ln(x)\cdot\frac{1}{x} = \frac{2\ln(x)\cos\left((\ln(x))^2\right)}{x} \quad \square$$

3. $y = \left(2e^{3x}+3e^{-2x}\right)^4$ *[1.5]*

SOLUTION. Power Rule, Chain Rule, Sum Rule, and "Multiplication by Constants":

$$\frac{dy}{dx} = \frac{d}{dx}\left(2e^{3x}+3e^{-2x}\right)^4 = 4\left(2e^{3x}+3e^{-2x}\right)^3\cdot\frac{d}{dx}\left(2e^{3x}+3e^{-2x}\right)$$
$$= 4\left(2e^{3x}+3e^{-2x}\right)^3\left(2\left[\frac{d}{dx}e^{3x}\right]+3\left[\frac{d}{dx}e^{-2x}\right]\right)$$
$$= 4\left(2e^{3x}+3e^{-2x}\right)^3\left(2e^{3x}\left[\frac{d}{dx}(3x)\right]+3e^{-2x}\left[\frac{d}{dx}(-2x)\right]\right)$$
$$= 4\left(2e^{3x}+3e^{-2x}\right)^3\left(2e^{3x}\cdot 3+3e^{-2x}(-2)\right) = 4\left(2e^{3x}+3e^{-2x}\right)^3\left(6e^{3x}-6e^{-2x}\right)$$
$$= 24\left(2e^{3x}+3e^{-2x}\right)^3\left(e^{3x}-e^{-2x}\right) \quad \square$$

4. $y = \sin\left(x^2\right)\cos\left(x^2\right)$ *[1.5]*

SOLUTION 1. Product Rule, Chain Rule, Power Rule, and a trigonomteric identity:

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\left(\sin\left(x^2\right)\cos\left(x^2\right)\right) = \left[\frac{d}{dx}\sin\left(x^2\right)\right]\cos\left(x^2\right) + \sin\left(x^2\right)\left[\frac{d}{dx}\cos\left(x^2\right)\right] \\
&= \left[\cos\left(x^2\right)\frac{d}{dx}x^2\right]\cos\left(x^2\right) + \sin\left(x^2\right)\left[\sin\left(x^2\right)\frac{d}{dx}x^2\right] \\
&= \left[\cos\left(x^2\right)\cdot 2x\right]\cos\left(x^2\right) + \sin\left(x^2\right)\left[-\sin\left(x^2\right)\cdot 2x\right] \\
&= 2x\cos^2\left(x^2\right) - 2x\sin^2\left(x^2\right) = 2x\left(\cos^2\left(x^2\right) - \sin^2\left(x^2\right)\right) = 2x\cos\left(2x^2\right) \quad \square
\end{aligned}$$

SOLUTION 2. A trigonometric identity, "Multiplication by Constants", Chain Rule, and Power Rule:

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\left(\sin\left(x^2\right)\cos\left(x^2\right)\right) = \frac{d}{dx}\left(\frac{1}{2}\sin\left(2x^2\right)\right) = \frac{1}{2}\cdot\frac{d}{dx}\sin\left(2x^2\right) \\
&= \frac{1}{2}\cos\left(2x^2\right)\cdot\frac{d}{dx}\left(2x^2\right) = \frac{1}{2}\cos\left(2x^2\right)\cdot 2\frac{d}{dx}\left(x^2\right) = \cos\left(2x^2\right)\cdot 2x \\
&= 2x\cos\left(2x^2\right) \quad \blacksquare
\end{aligned}$$