Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Fall 2023

Final Examination
15:00-18:00 in SC 137 on Wednesday, 20 December.

Instructions: Do both of parts **X** and **Y**, and, if you wish, part **Z**. Please show all your work, justify all your answers, and simplify these where you reasonably can. When you are asked to do k of n questions, only the first k that are not crossed out will be marked. *If you have a question, or are in doubt about something,* **ask!**

Aids: Any calculator, as long as it can't communicate with other devices; (all sides of) one letter- or A4-size sheet; one brain (no neuron limit).

Part X. Do all four (4) of **1**–**4**.

1. Compute $\dfrac{dy}{dx}$ as best you can in any four (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. $y = e^{x^2+1}$ **b.** $y = xe^{-x}$ **c.** $y = \ln(\cos(x))$

d. $y = \left(x^3+41\right)^{13}$ **e.** $y = \dfrac{x}{1+x^2}$ **f.** $y = \displaystyle\int_{\sqrt{\pi}}^{\cos(x)} t\,dt$

2. Evaluate any four (4) of the integrals **a**–**f**. *[20 = 4 × 5 each]*

a. $\displaystyle\int_0^1 \frac{x+1}{x^2+2x+1}\,dx$ **b.** $\displaystyle\int x\sec^2(x)\,dx$ **c.** $\displaystyle\int \tanh(x)\,dx$

d. $\displaystyle\int_0^{\pi/2} \sin(2x)\,dx$ **e.** $\displaystyle\int_0^1 \frac{x^2}{e^x}\,dx$ **f.** $\displaystyle\int \frac{\arctan(x)}{1+x^2}\,dx$

3. Do any four (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. Use the ε–δ definition of limits to verify that $\displaystyle\lim_{x\to 1}(3x-5) = -2$.

b. At what value(s) of x, if any, does the graph of $y = \dfrac{x}{1+x^2}$ have a tangent line with slope 1?

c. Compute $\displaystyle\lim_{x\to\infty} \frac{x^2+1}{e^x+1}$.

d. Find $g(x)$ if $g'(x) = \cos(\pi x)$ and $g(1) = \dfrac{1}{\pi}$.

e. Let $h(x) = \begin{cases} e^{-x^2} & x \neq 0 \\ 0 & x = 0 \end{cases}$. Determine whether $h(x)$ is continuous at $x = 0$.

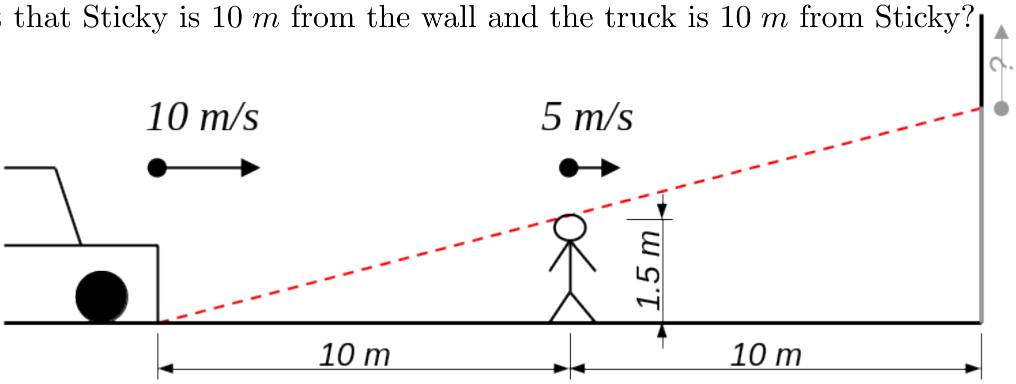
f. Sketch the finite region between $y = x+1$ and $y = 2^x$, for $0 \le x \le 1$, and find its area.

4. Find the domain, intercepts, vertical and horizontal asymptotes, intervals of increase and decrease, maximum and minimum points, intervals of concavity, and inflection points of $f(x) = \dfrac{e^x}{e^x+1}$. *[12]*

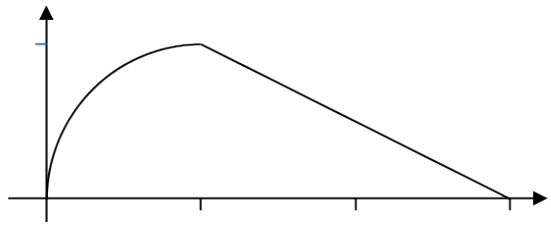
*Sadly, there is more: Parts **Y** and **Z** are on page 2.*

Part Y. Do any two (2) of **5**–**7**. *[28 = 2 × 14 each]* *. . . and here is "more"!*

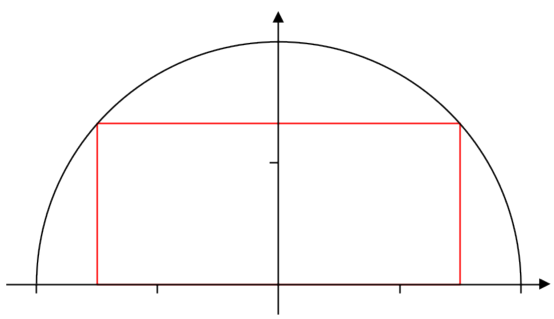
5. Sticky is trapped in a dead-end alley! An extreme low-rider truck with its headlights at ground level is chasing sticky down the alley, straight towards the wall at the end of the alley. Sticky, who is 1.5 m tall, is running towards the wall at a constant speed of 5 m/s and the truck is driving towards Sticky and the wall at a constant speed of 10 m/s. Sticky casts a shadow in the light from the truck headlights upon the wall at the end of the alley. How is the tip of Sticky's shadow moving on the wall at the instant that Sticky is 10 m from the wall and the truck is 10 m from Sticky?



6. The region between the x-axis and $y = \sqrt{x}$, for $0 \leq x \leq 1$, and between the x-axis and $y = -\dfrac{x}{2} + \dfrac{3}{2}$, for $1 \leq x \leq 3$, is revolved about the x-axis. Sketch the resulting solid and find its volume.



7. Find the maximum possible area of a rectangle whose base is on the x-axis and whose upper corners are on the semi-circle $y = \sqrt{4 - x^2}$.



[Total = 100]

Part Z. *Here be bonus points!* Do one or both of **7 + 1** and **7 + 2**.

7 + 1. Give a funny and clever way for Sticky to get out of the sticky situation described in question **5**. *[1]*

7 + 2. Does Euler's polynomial $x^2 - x + 41$ always give you a prime number when x is a positive integer? If it does, explain why; if not, give an example where it doesn't. *[1]*

I HOPE THAT YOU ENJOYED THE COURSE. ENJOY THE BREAK EVEN MORE!