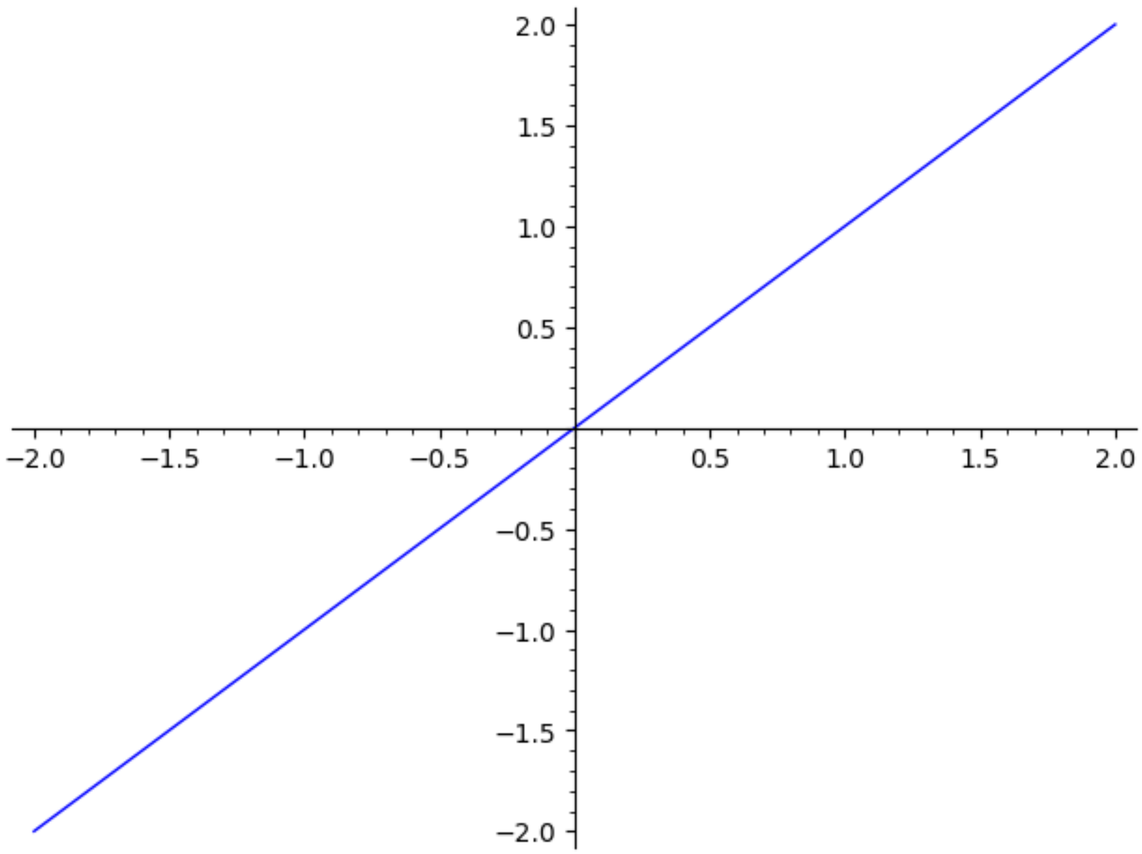
In [1]:

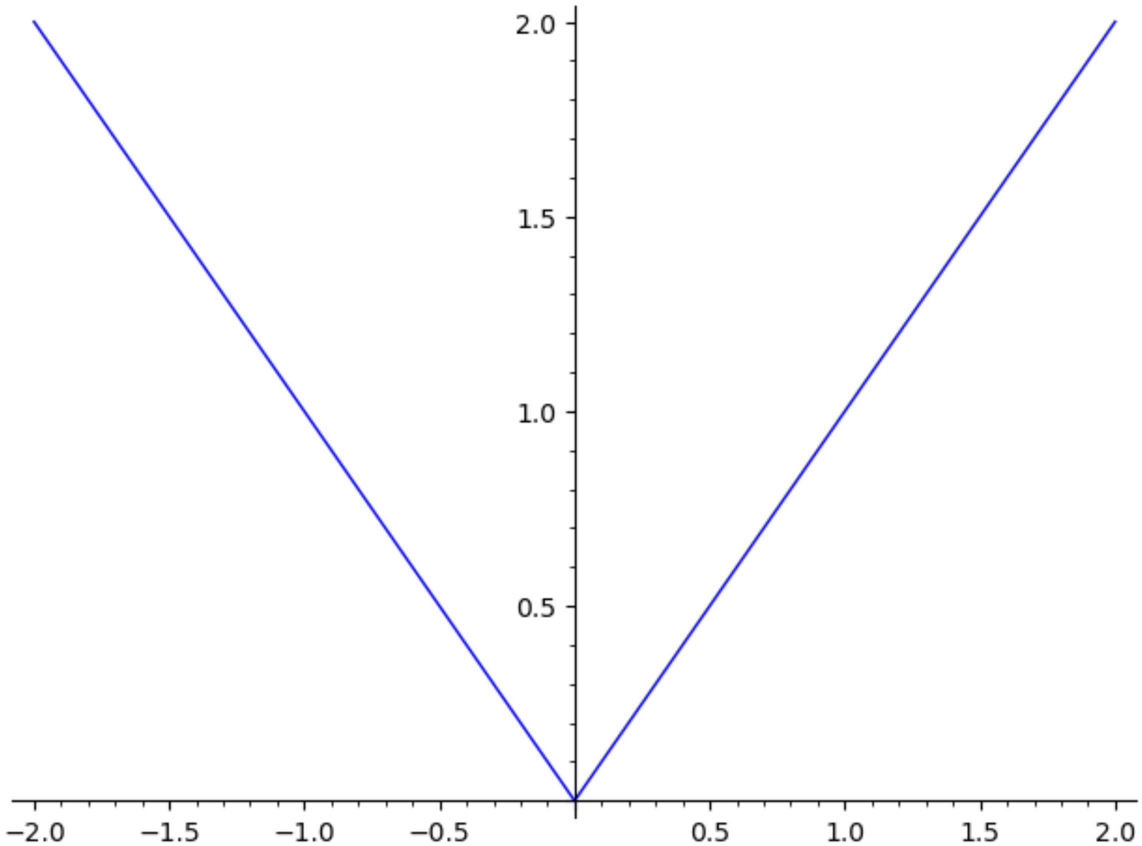
```
# MATH 1110H,  Fall 2023
# Solutions to Assignment #1
#
# 1. Plot the following graphs in Cartesian coordinates.
#
# 1a. y = x for -2 <= x <= 2.

plot(x, -2, 2)
```

Out[1]:



In [2]:
```
# 1b.  y = |x| for -2 <= x <= 2.

plot(abs(x), -2, 2)
```

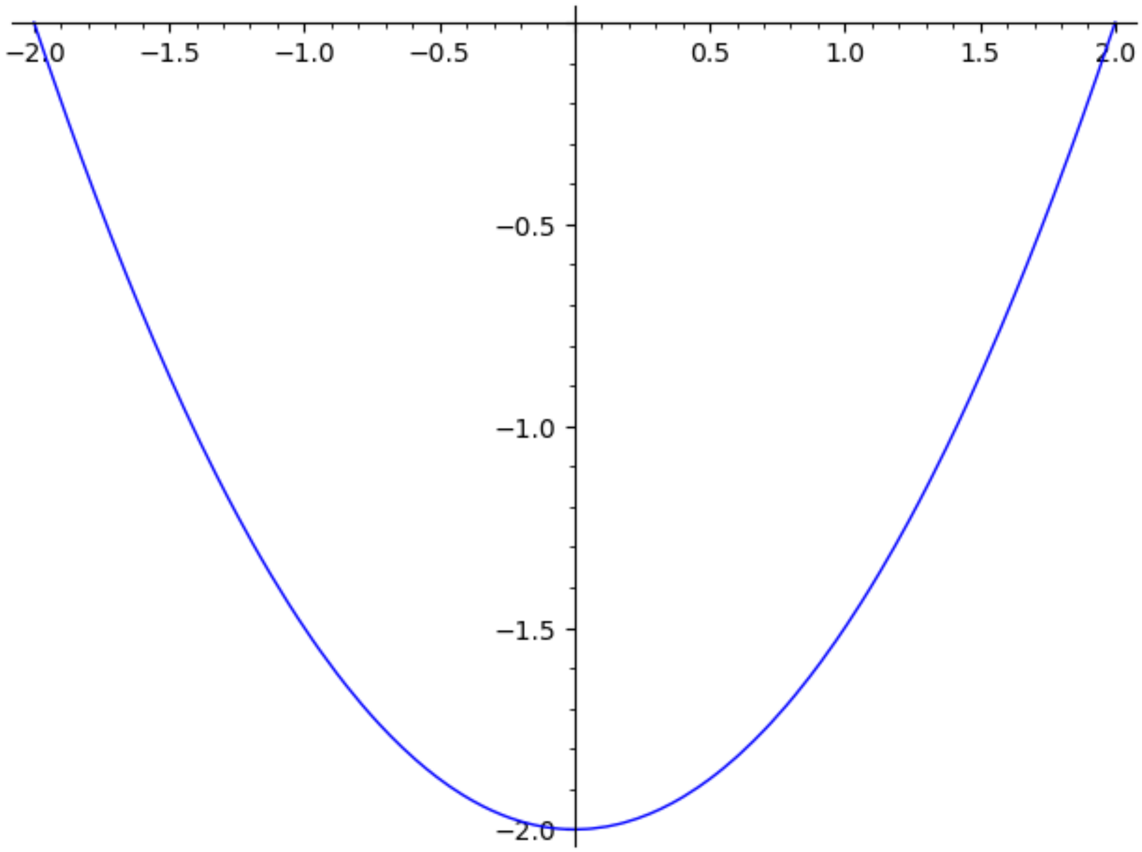
Out[2]:



In [3]:

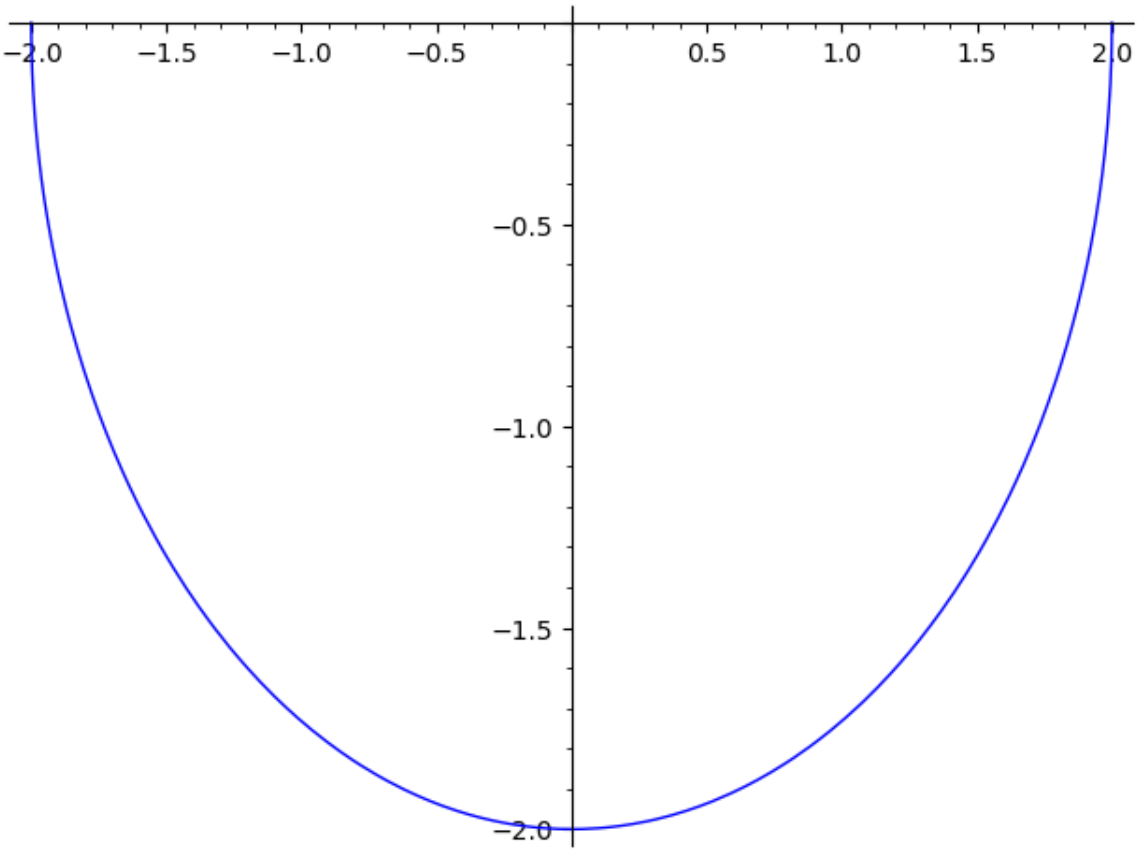
```
# 1c.  y = (x^2 - 4)/2 for -2 <= x <= 2.

plot((x^2 - 4)/2, -2, 2)
```

Out[3]:



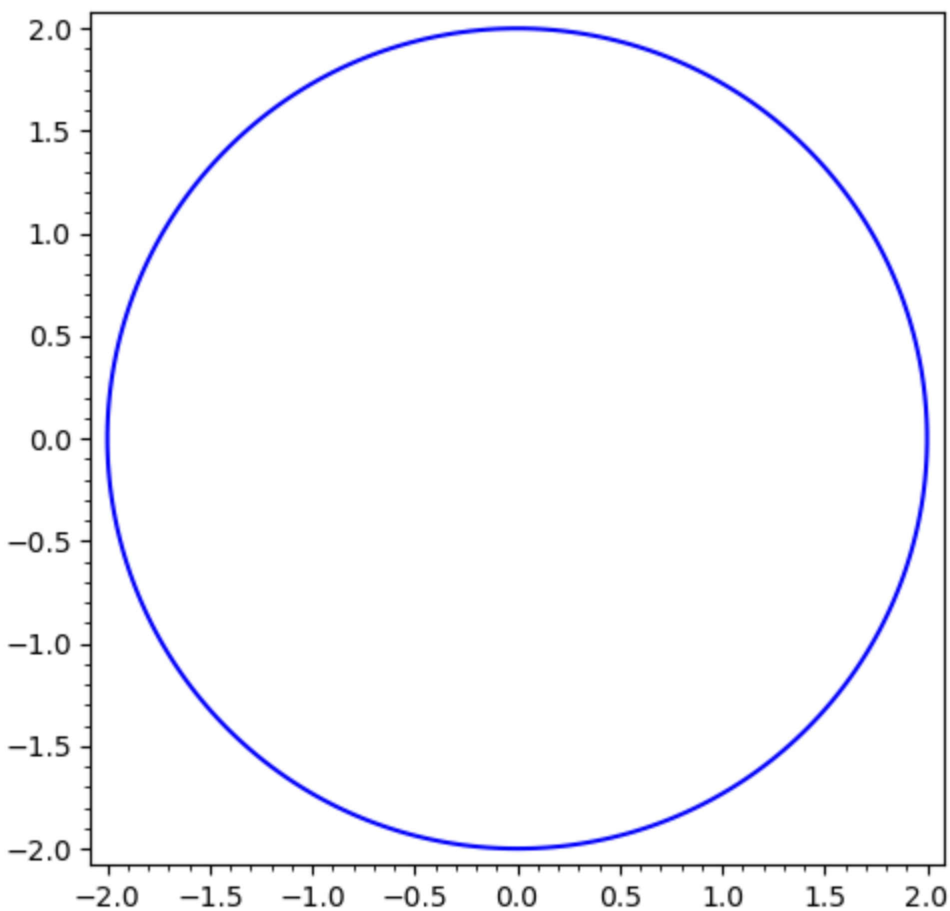
In [4]:
```
# 1d.  y = -sqrt(4-x^2) for -2 <= x <= 2.

plot(-sqrt(4-x^2), -2, 2)
```

Out[4]:



In [5]:
```
# 2.  Plot the following implicitly defined curves.
#
# 2a.  x^2 + y^2 = 4 for all x and y for which this equation makes
#      sense.

var("y")  # Since x is the only thing assumed to be a variable.

implicit_plot(x^2 + y^2 == 4, (x,-2,2), (y,-2,2))

# Note that implicit_plot wants you to specify the desired ranges
# for both variables,  so if you want to plot all those that make
# sense,  you need to work those ranges out or experiment a bit.
# Note also the use of == tor equality as a relation,  rather than
# =,  which is an assignment operator in SageMath
```

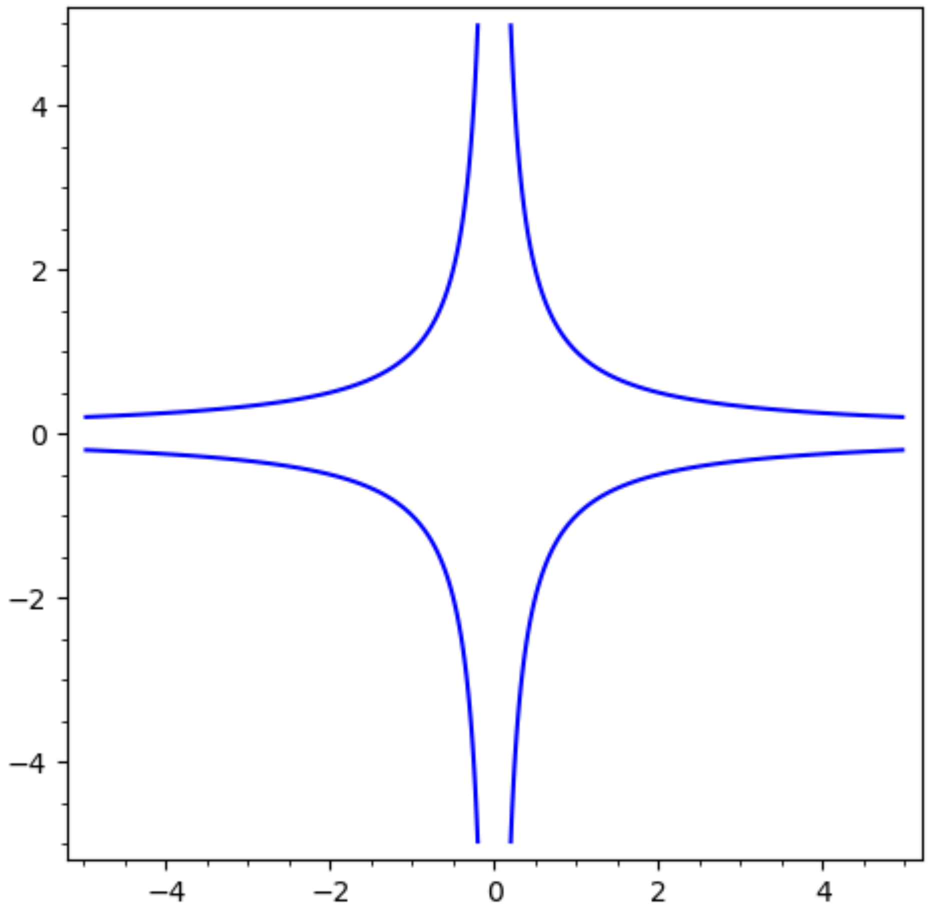
Out[5]:



In [6]:

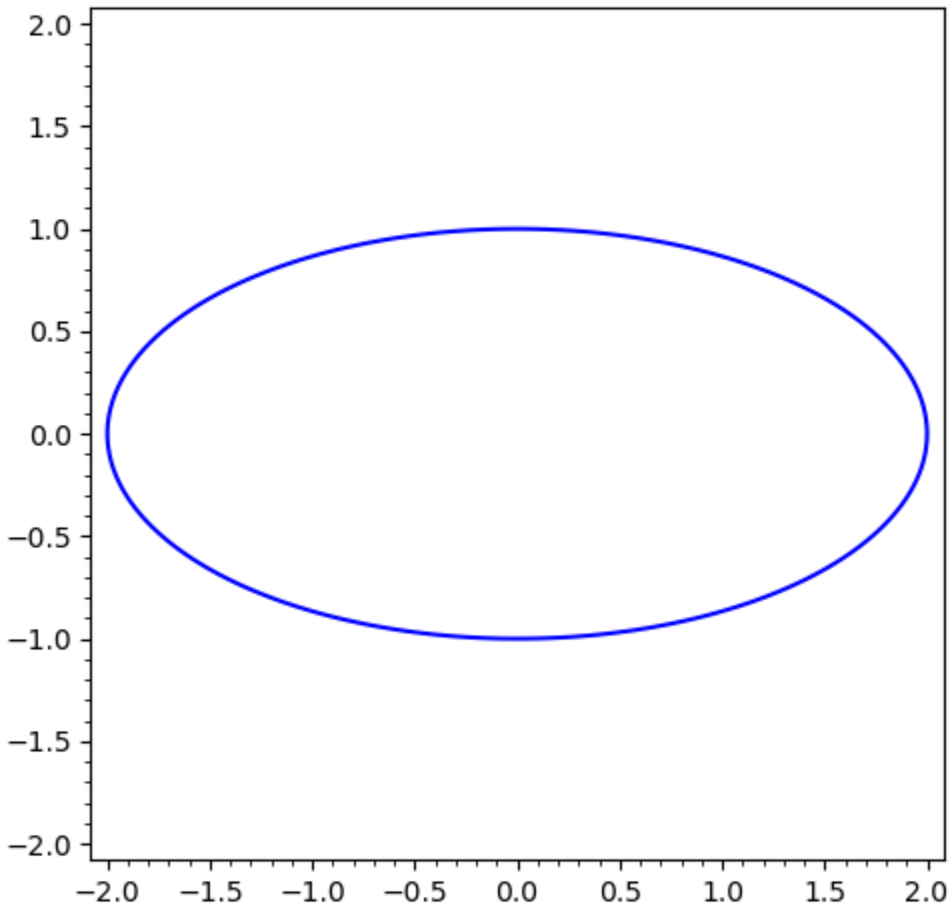
```
# 2b.  |xy|=1 for all x and y for which this equation makes sense.

implicit_plot(abs(x*y) == 1, (x,-5,5), (y,-5,5))

# In this case,  there is no way to display the plot for all x and
# y, as the possible values include all real numbers except 0.
# Note also that multiplication must be explicitly specified.
```

Out[6]:



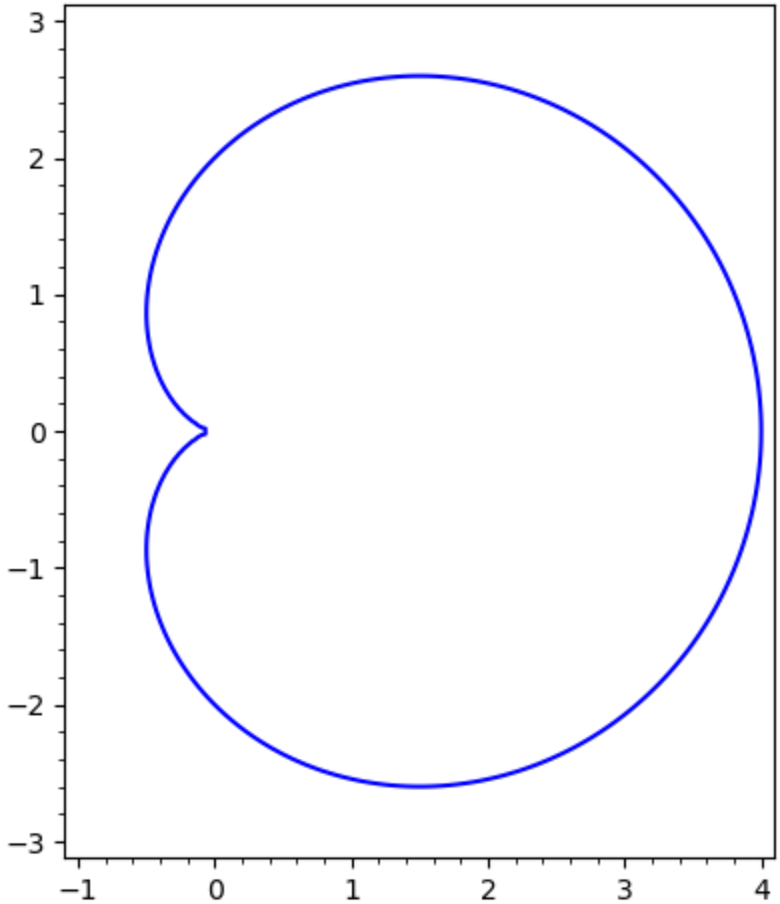
In [7]:
```
# 2c.  x^2 + 4y^2 = 4 for all x and y for which this equation makes
#      sense.

implicit_plot(x^2 + 4*y^2 == 4, (x,-2,2), (y,-2,2))
```

Out[7]:



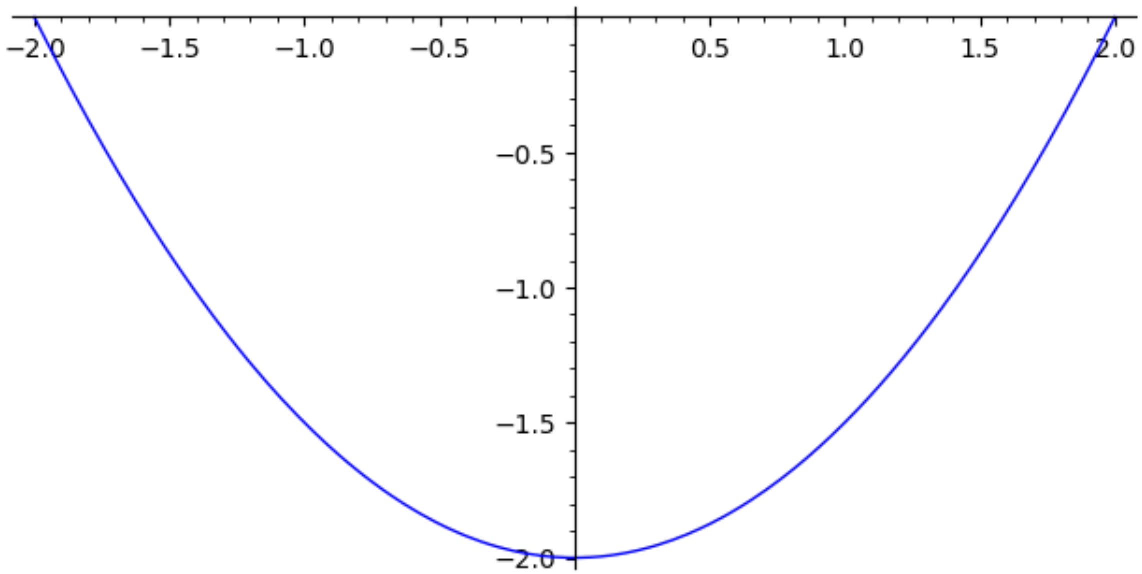
In [8]:
```
# 2d.  (x^2 + y^2)^2 - 4x(x^2 + y^2) - 4y^2 = 0 for all x and y for
#      which this equation makes sense.

implicit_plot((x^2 + y^2)^2 - 4*x*(x^2 + y^2) - 4*y^2 == 0, (x,-1,4),
```

Out[8]:



In [9]:
```
# 3.  Plot the following parametric curves.
#
# 3a.  x = 2t and y = 2t^2 - 2 for -1 <= t <= 1.

var("t") # Since x is the only thing assumed to be a variable.

parametric_plot((2*t, 2*t^2 - 2), (t,-1,1))
```

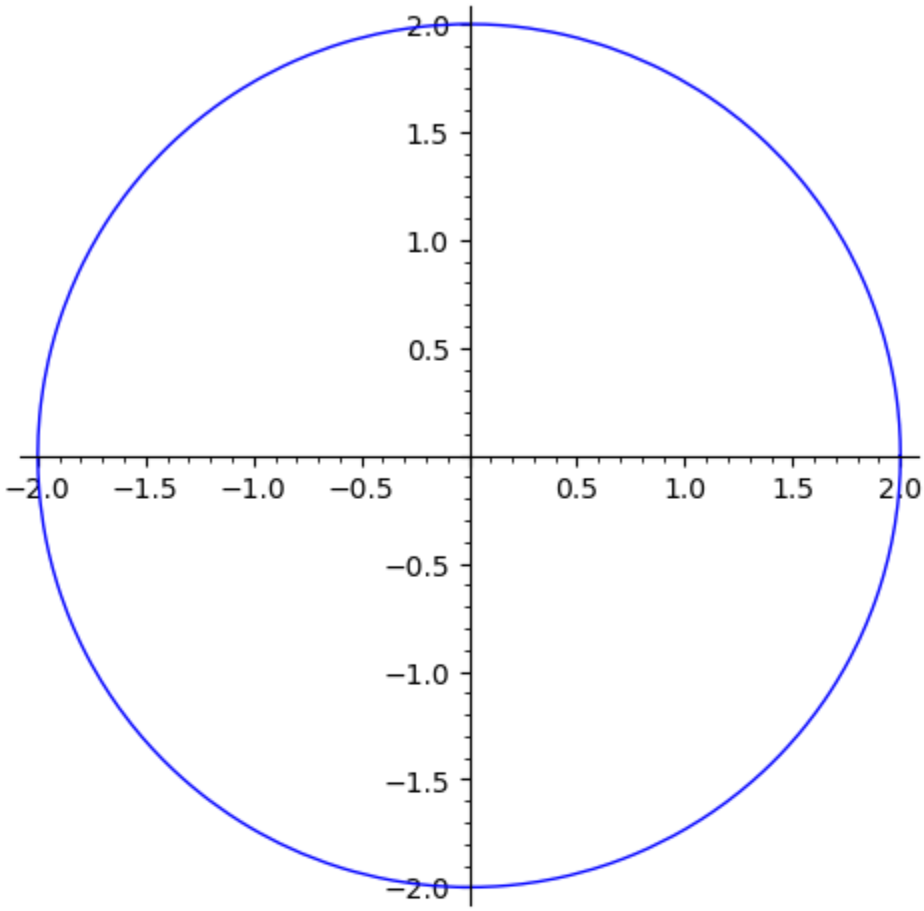
Out[9]:



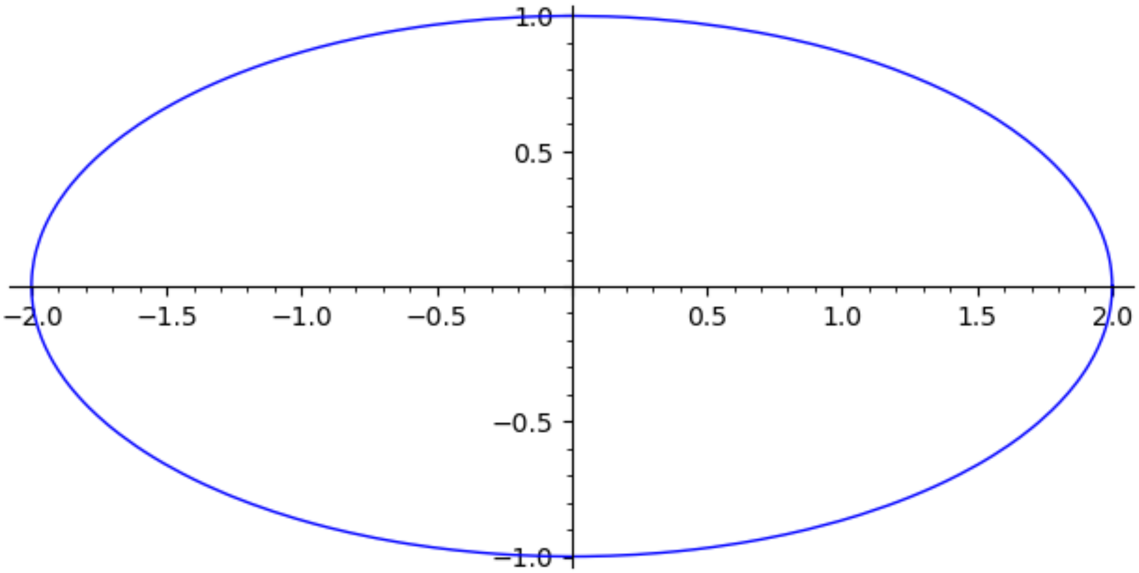
In [10]:

```
# 3b.  x = 2cos(t) and y = 2sin(t) for 0 <= t <= 2pi.

parametric_plot((2*cos(t), 2*sin(t)), (t,0,2*pi))

# Note that the name of that popular constant in Sagemath is just pi.
```

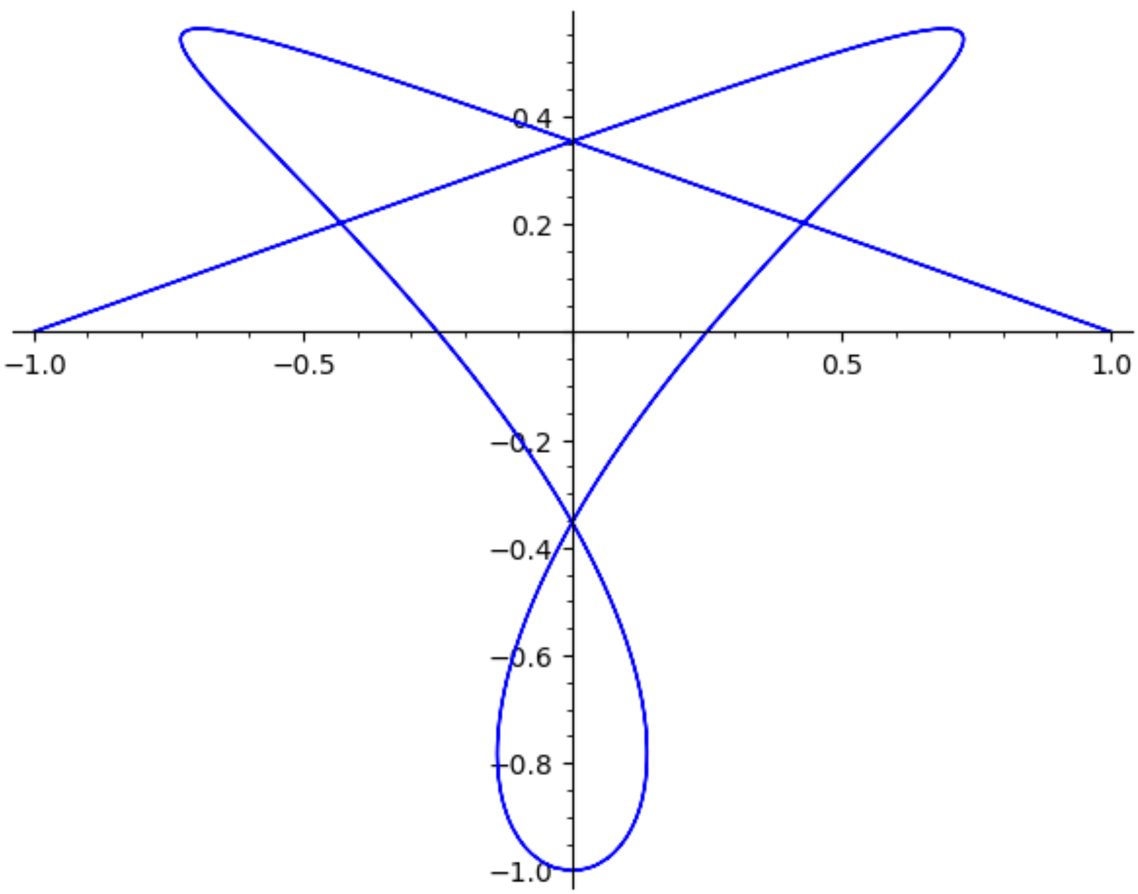
Out[10]:



In [11]:
```
# 3c.  x = 2cos(t) and y = sin(t) for 0 <= t <= 2pi.

parametric_plot((2*cos(t), sin(t)), (t,0,2*pi))
```
Out[11]:

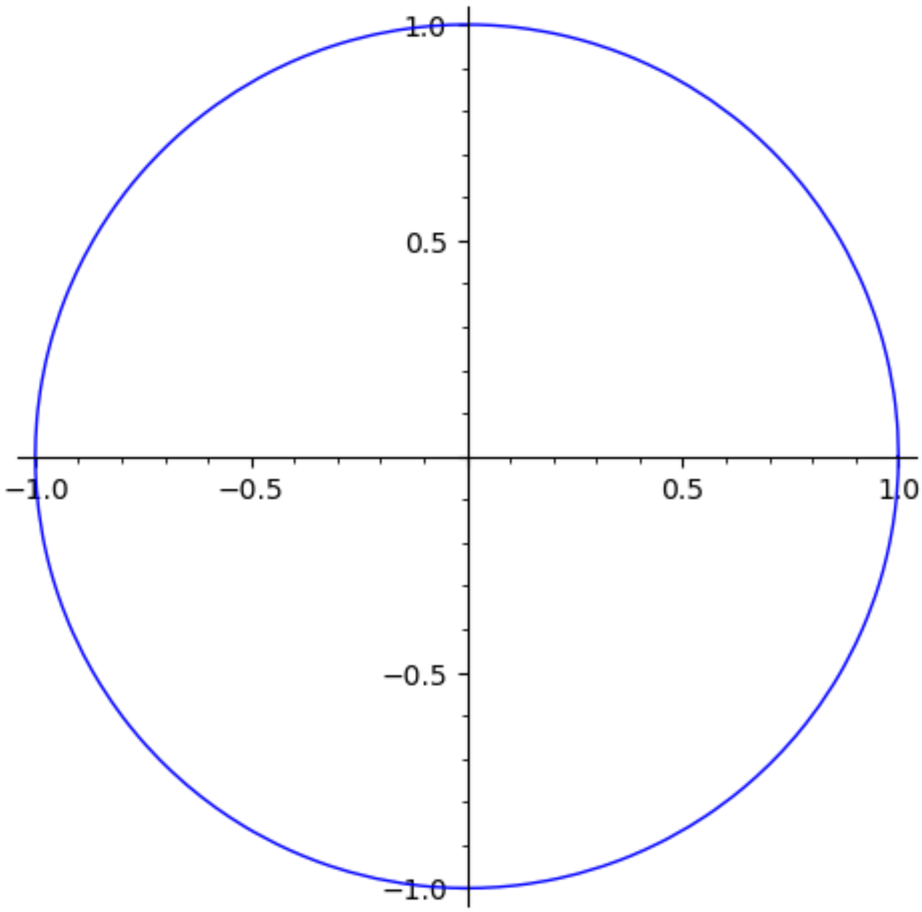


In [12]:
```
# 3d.  x = cos(4t)cos(t) and y = sin(3t)sin(t) for 0 <= t <= 2pi.

parametric_plot((cos(4*t)*cos(t), sin(3*t)*sin(t)), (t,0,2*pi))
```
Out[12]:



In [13]:
```
# 4.  Plot the following curves in polar coordinates.
#
# 4a. r = 1 for 0 <= theta <= 2pi.

var("theta")  # Since x is the only thing assumed to be a variable.

polar_plot(1, (theta,0,2*pi))
```

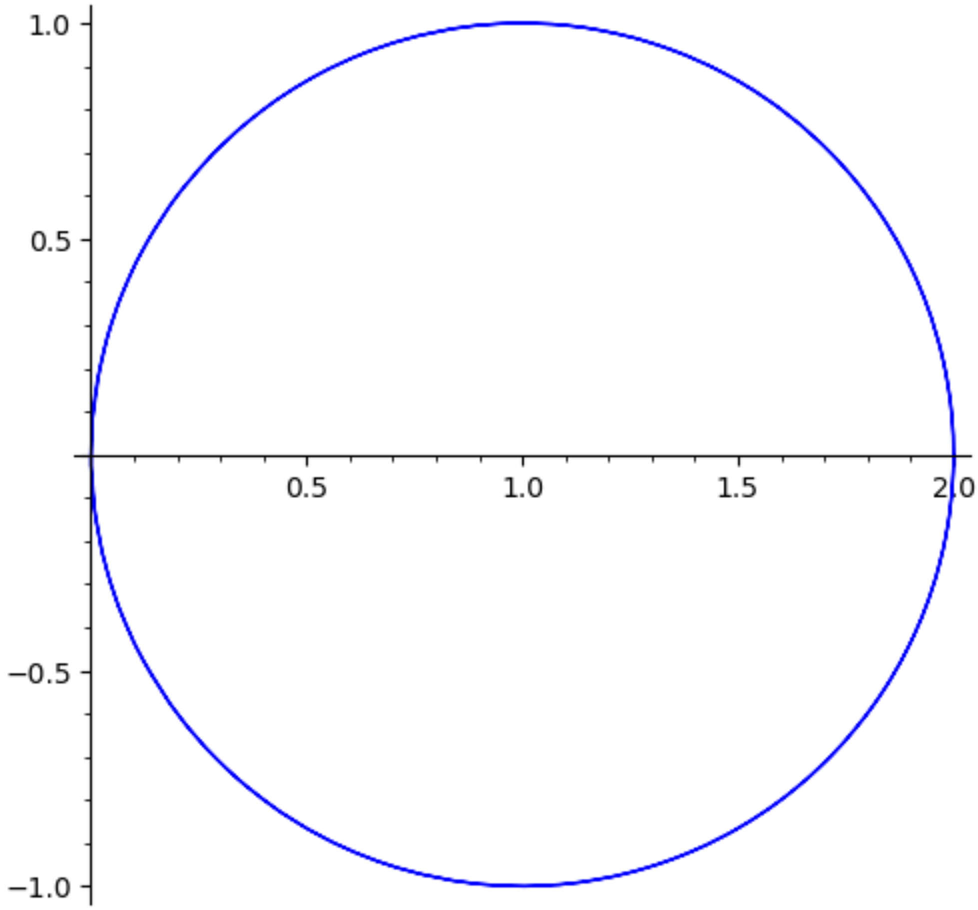
Out[13]:



In [14]:

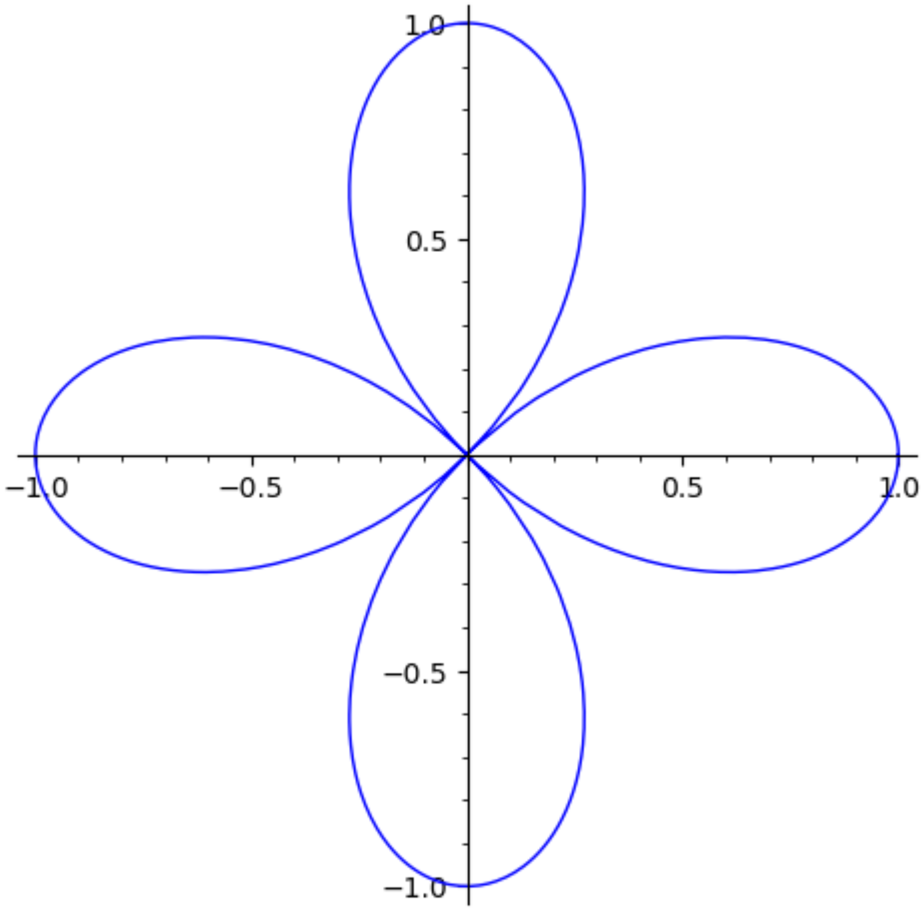
```
# 4b.  r = 2cos(theta) for 0 <= theta <= 2pi.

polar_plot(2*cos(theta), (theta,0,2*pi))
```

Out[14]:



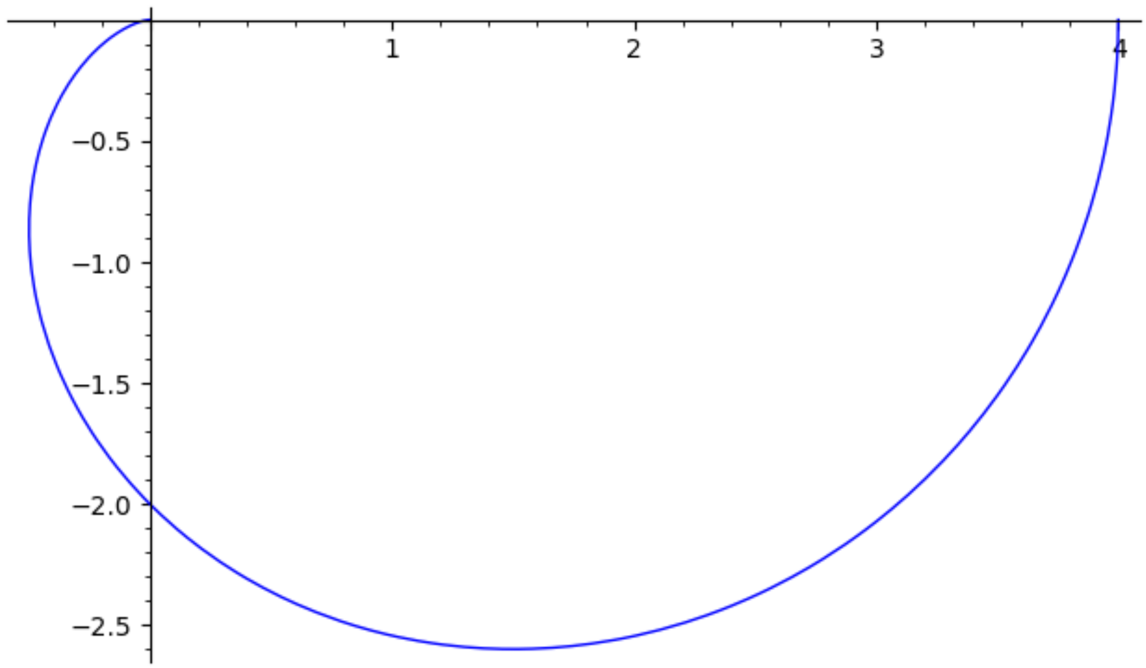
In [15]:
```
# 4c.  r = cos(2*theta) for 0 <= theta <= 2pi.

polar_plot(cos(2*theta), (theta,0,2*pi))
```

Out[15]:



In [16]:
```
# 4d.  r = 2(cos(theta) - 1) for 0 <= theta <= pi.

polar_plot(2*(cos(theta)-1), (theta,0,pi))
```

Out[16]:



In [17]:
```
# 5.  Some of the curves in questions 1-4 are part or all of other
#     curves in questions 1-4.  Identify as many such cases as you
#     can.
#
# Solution.  Here are the cases where one of the curves in questions
# 1-4 is all or part of another curve in questions 1-4:
#
# i.    The piece of a parabola in 1c is the same curve as 3a.
#
# ii.   The semi-circle in 1d is the lower half of the circles in 2a
#       and 3b.
#
# iii. The circles in 2a and 3b are both the circle of radius 2 with
#       centre at the origin.  Note that neither is the circle in 4a
#       or the one in 4b, as both of these circles have radius 1.
#       (The circles in 4a and 4b are not the same either: they have
#       different centres.)
#
# iv.   The curves in 2c and 3c are the same ellipse.
#
# v.    The curve in 4d is the lower half of the cardioid in 2d.
#
# I think that's all! :-)
```