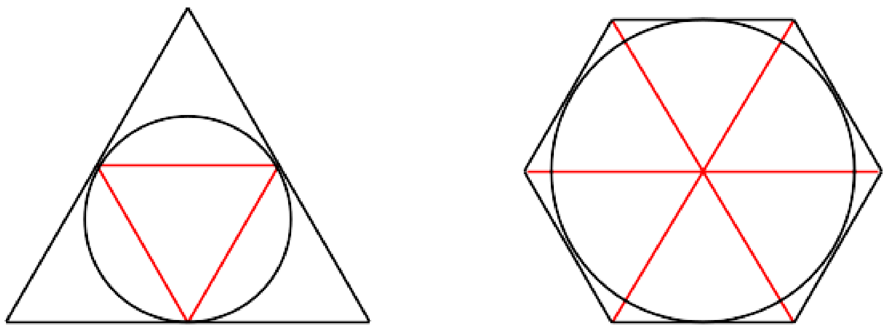
Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals (Section C)
TRENT UNIVERSITY, Fall 2021

Solution to Quiz #7
A puzzling change of pace!
Wednesday, 17 November.

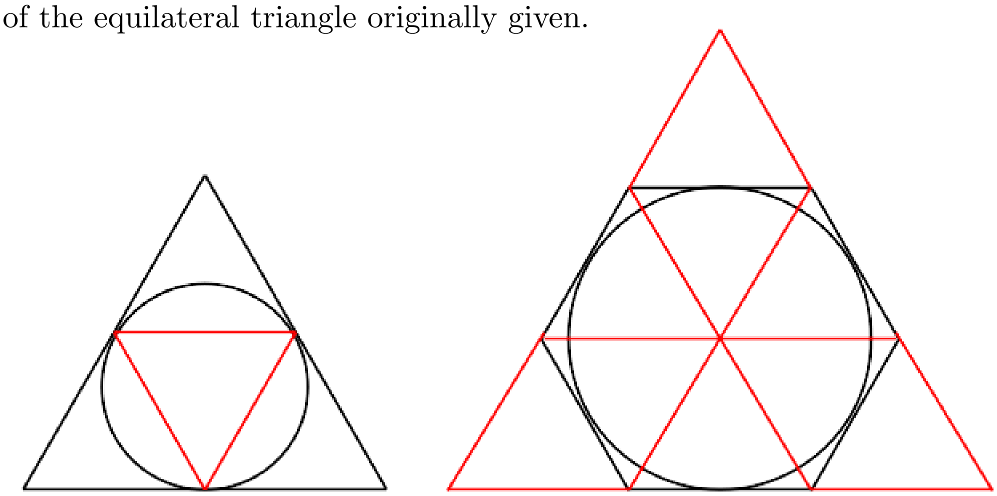
1. Suppose one is given twelve line segments of equal length, of which six are used to make an equilateral triangle and six are used to make a regular hexagon. Circles are inscribed inside each polygon, touching each side of its respective polygon at the midpoint only, as in the diagram below.



Find the ratio of the area of the circle inscribed in the triangle to the area of the circle inscribed in the hexagon. *[10]*

NOTE: This can be done without using calculus, or even trigonometry, though you are welcome to use any and all mathematical tools you wish.

SOLUTION. Observe that the equilateral triangle can be decomposed into four smaller equilateral triangles with side lengths equal to the lengths of the original twelve line segments, while the regular hexagon can be decomposed into six such triangles. (Drawn in above!) Consider the following diagram, in which three more such small triangles are added to the hexagon to turn it into an equilateral triangle with sides (and width and height) 1.5 times longer that of the equilateral triangle originally given.



Since the (similar!) triangles they are contained in are scaled in a $2:3$ ratio, the circles' radii are scaled in the same ratio. It follows that the larger circle, originally inscribed in the hexagon, has $\left(\frac{3}{2}\right)^2 = \frac{9}{4} = 2.25$ the area of the smaller one; equivalently, the smaller one has $\left(\frac{2}{3}\right)^2 = \frac{4}{9} = 0.\dot{4} = 0.4444\ldots$ the area of the larger one. ■

Hope you enjoyed the change of pace!