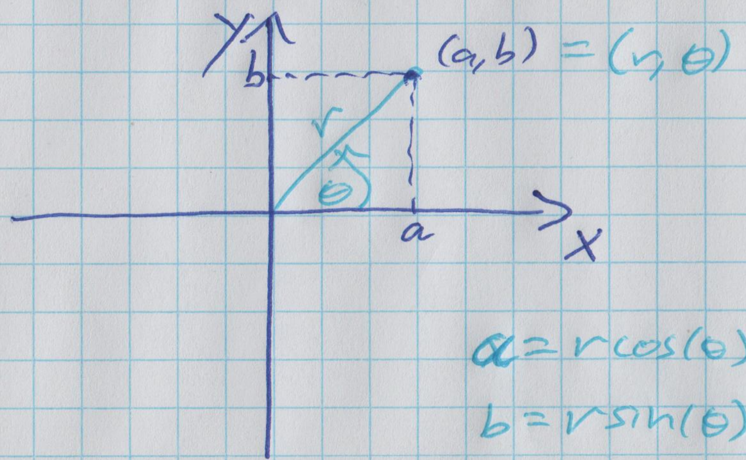
Welcome to MATH 1110H: Calculus I

Today: Some background and notation

Calculus in the child of geometry and algebra, united by having a coordinate system.

Cartesian coordinates

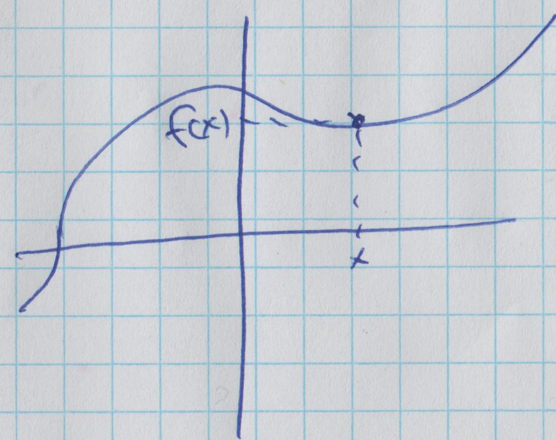


$a = r\cos(\theta)$

$b = r\sin(\theta)$

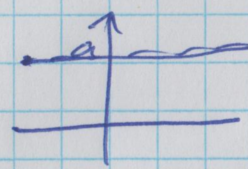
Polar coordinates

$y = f(x)$



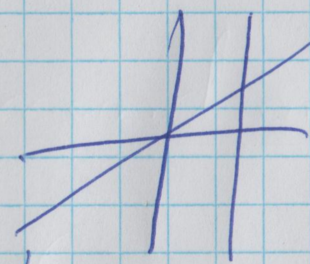
Functions we'll need:

0. constant functions



$y = a$

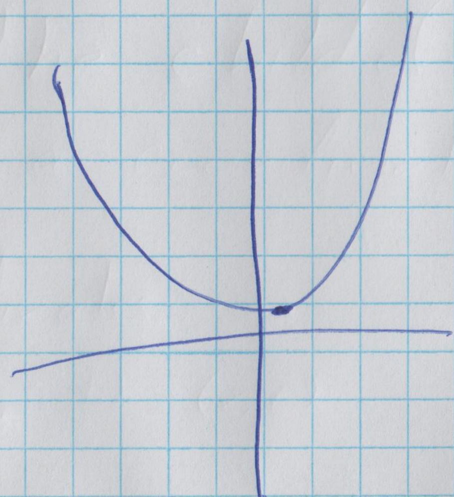
1. linear functions



$y = ax + b$

or $x = a$

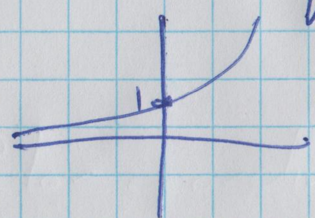
2. parabolas



$y = ax^2 + bx + c$

3. exponential functions $f(x) = a^x$ $(a > 0)$

Especially the "natural" exponential function



$y = e^x$

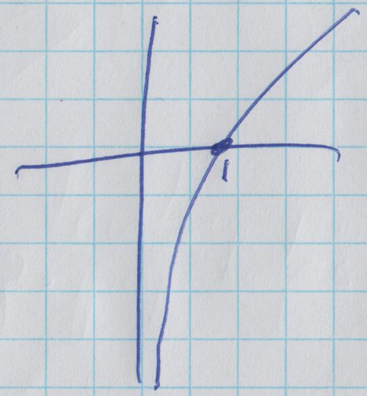
↑ Euler's constant $e \approx 2.718\ldots$

4. Logarithmic functions (inverse functions of the exponential functions)

$y = \log_a(x)$ $(a>0, x>0)$

$\Leftrightarrow e^y = x$

$$\left[\begin{array}{l} f(f^{-1}(x)) = x \\ f^{-1}(f(x)) = x \end{array}\right]$$



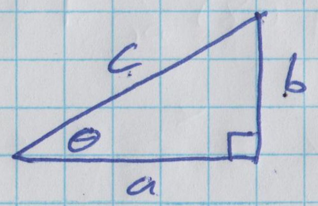
Natural logarithm ~~exponential~~ function

$\ln(x) = \log_e(x)$.

5. polynomial functions

$$y = a_k x^k + a_{k-1}^{k-1} + \cdots + a_1 x + a_0$$

6. Trig functions



$\sin(\theta) = \frac{b}{c}$

$\cos(\theta) = \frac{a}{c}$

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$

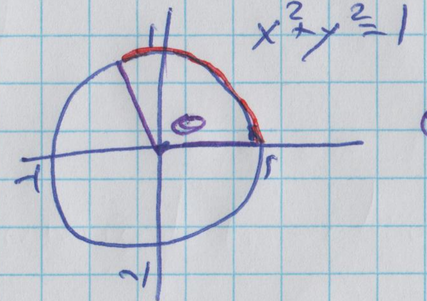
$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$

$\sec(\theta) = \frac{1}{\cos(\theta)}$

$\csc(\theta) = \frac{1}{\sin(\theta)}$

We'll usually measure angles in radians

Look up Formulas for Success for more.

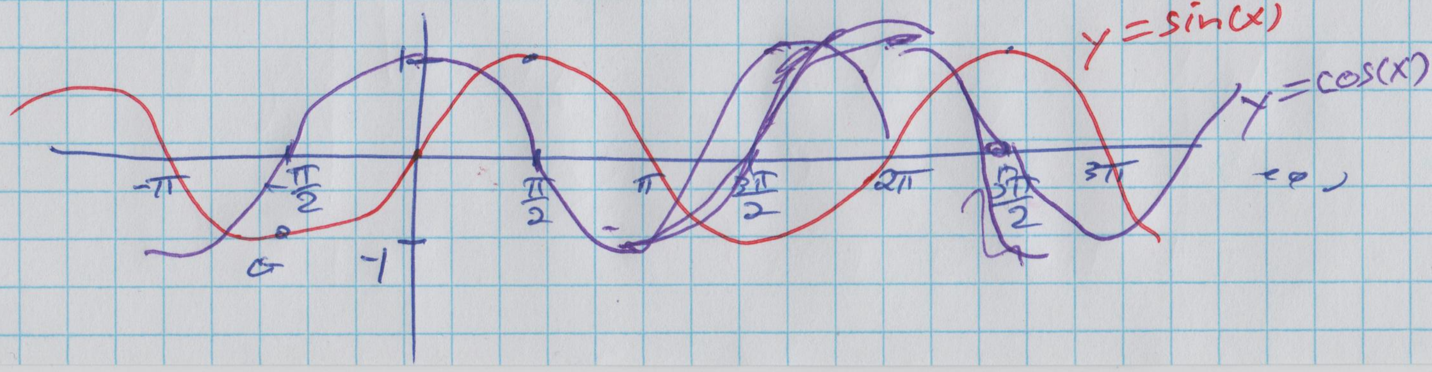


θ = arc length subtended by the angle in a unit circle.

$C = 2\pi r \quad (r=1)$
$= 2\pi$

∟ $= 90^\circ = \frac{\pi}{2}$ radians

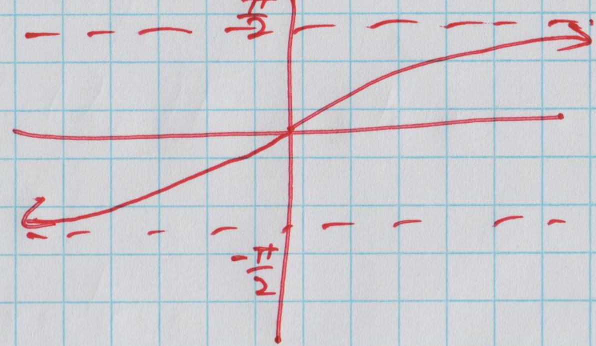
$60^\circ = \frac{\pi}{3} \quad 30^\circ = \frac{\pi}{6} \quad 45^\circ = \frac{\pi}{4} \ldots$



7. Inverse trig functions

I hate the notation $\tan^{-1}(x)$, $\sin^{-1}(x)$, $\cos^{-1}(x)$, ... for the inverse functions: too many people confuse them with $\cot(x)$, $\csc(x)$, $\sec(x)$, ... resp.

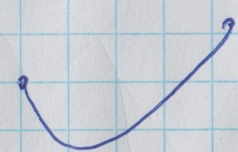
trig fn.	inverse trig fn.
$\sin(x)$	$\arcsin(x)$
$\cos(x)$	$\arccos(x)$
$\tan(x)$	$\arctan(x)$
⋮	⋮



8. Hyperbolic function

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

"sinch"



$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

"kosh"

Algebra

You need to be able to solve various algebraic problems:

eg solve a linear equation: $ax+b=0 \quad (a\neq 0)$

$$\Rightarrow ax=-b \Rightarrow x=\frac{-b}{a}$$

solve a quadratic equation: $ax^2+bx+c=0 \quad (a\neq 0)$

Use the quadratic formula: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Why is this so?

We'll see!

$(a \neq 0)$

$$ax^2+bx+c=0$$

$$a\left(x^2+\underbrace{\frac{b}{a}}_{2p}x+\frac{c}{a}\right) = a\left(\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\left(\frac{b}{2a}\right)^2\right)$$

$$\boxed{(x+p)^2 = x^2+2px+p^2}$$

$$= a\left(\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}\right)$$

$$= a\left(\left(x+\frac{b}{2a}\right)^2+\frac{-b^2+4ac}{4a^2}\right)$$

$$\Rightarrow a\left(\left(x+\frac{b}{2a}\right)^2+\frac{-b^2+4ac}{4a^2}\right)=0$$

$$\Rightarrow \left(x+\frac{b}{2a}\right)^2+\frac{-b^2+4ac}{4a^2}=0$$

$$\Rightarrow \left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2} \Rightarrow x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$\Rightarrow x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad \checkmark$$

Inequalities

$a \leq b \quad \Rightarrow \quad \frac{1}{a} \geq \frac{1}{b} \quad \& \quad a+c \leq b+c$

$(a \neq 0, b \neq 0)$

$\& \quad -a \geq -b \quad \ldots$

Geometric mean of a & b: $\sqrt{ab}$

Arithmetic mean of a & b: $\frac{a+b}{2}$

If $a \geq 0$ & $b \geq 0$, then $\sqrt{ab} \leq \frac{a+b}{2}$.

Here's the proof: $(a-b)^2 \geq 0$

$$\| $$

$$a^2 - 2ab + b^2 \quad \Rightarrow \quad a^2+b^2 \geq 2ab$$

$$\Rightarrow \quad \frac{a^2+b^2}{2} \geq ab = \sqrt{a^2b^2}$$

So we replace a^2 by a (or a by $\sqrt{a}$) & b^2 by b (or b by $\sqrt{b}$)

So: $\boxed{\frac{(\sqrt{a})^2 + (\sqrt{b})^2}{2} = \frac{a+b}{2} \geq \sqrt{(\sqrt{a})^2(\sqrt{b})^2} = \sqrt{ab}}$

Absolute values

$|x| =$ distance x is from 0

$$= \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x \leq 0 \end{cases}$$

Basic properties: $|ab| = |a| \cdot |b|$

$|a+b| \leq |a| + |b|$ ($\&$ $=$ only if both $a, b \geq 0$ or both $a, b \leq 0$)

eg $|-2x+3| = 1$

$$\Rightarrow -2x+3 = \pm 1 \Rightarrow -2x = -3 \pm 1 \Rightarrow x = \frac{-3 \pm 1}{-2}$$

ie $x = \frac{-3+1}{-2} = \frac{-2}{-2} = 1$ or $x = \frac{-3-1}{-2} = \frac{-4}{-2} = 2$

Next time: limits