Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Fall 2019
Final Examination for Section A

Space-time: *Gym – 19:00-22:00 on Saturday, 14 December, 2019.*

Instructions: Do parts **X** and **Y**, and, if you wish, part **Z**. Please show all your work and justify all your answers. *If in doubt about something,* **ask!**

Aids: Any calculator; (all sides of) one aid sheet; one cranial neural net.

Part X. Do *all four* (4) of **1**–**4**. *[Subtotal = 74]*

1. Compute $\frac{dy}{dx}$ as best you can in any *four* (4) of **f**–**a**. *[20 = 4 × 5 each]*

a $y = \dfrac{x^2}{x+1}$ **b.** $y = \displaystyle\int_0^{\cos(x)} t^2\,dt$ **c.** $y = \displaystyle\int_0^1 13y^{17\pi-1}\,dy$

d. $y = \left(e^{x+1}\right)^3$ **e.** $y = (x+1)\ln(x)$ **f.** $y = \arctan\left(\sqrt{x}\right)$

2. Evaluate any *four* (4) of the integrals **f**–**a**. *[20 = 4 × 5 each]*

a. $\displaystyle\int 2^{3x}\,dx$ **b.** $\displaystyle\int \frac{1+t}{1+t^2}\,dt$ **c.** $\displaystyle\int z\ln\left(z^2\right)\,dz$

d. $\displaystyle\int_1^e \frac{\ln(w)}{2w}\,dw$ **e.** $\displaystyle\int \frac{2\arctan^2(2v)}{1+4v^2}\,dv$ **f.** $\displaystyle\int_2^e \left(2u+e^u\right)\,du$

3. Do any *four* (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. Find the area of the the region between $y = x^3$ and $y = x$ for $-1 \le x \le 1$.

b. Compute $\lim_{x\to-\infty} xe^x$.

c. Find the equation of the tangent line to $y = e^{x-1}$ at $x = 1$.

d. Use the limit definition of the derivative to show that $\frac{d}{dx}\left(x^2+1\right) = 2x$ for all x.

e. Find the maximum value of $f(x) = x + \cos(x)$ on the interval $[0, \pi]$.

f. Use the ε–δ definition of limits to verify that $\lim_{x\to 2}(4-x) = 2$.

4. Find the domain and any and all intercepts, vertical and horizontal asymptotes, intervals of increase, decrease and concavity, and maximum, minimum, and inflection points of $f(x) = \left(\dfrac{x+1}{x}\right)^2 = \dfrac{x^2+2x+1}{x^2}$, and sketch its graph. *[14]*

More questions on page 2!

Part Y. Do any *two* (2) of **5–7**. *[Subtotal = 26 = 2 × 13 each]*

5. Determine the maximum possible area of a rectangle with two corners on the x-axis and the other two corners on the part of the parabola $y = 3 - x^2$ that is above the x-axis.

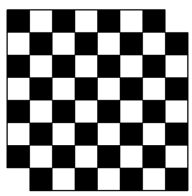
6. Find the volume of the solid obtained by revolving the triangle with vertices at $(0, 0)$, $(1, 0)$, and $(1, 1)$ about, respectively, **a.** the y-axis *[6]* and **b.** the x-axis *[7]*. Sketch each of the solids.

7. The top of a 6 m extension ladder rests against a vertical wall with its base resting on the horizontal floor 2 m from the wall. Something knocks the base of the ladder loose and it begins to slide away from the wall at a constant rate of 0.5 m/s. The knock also loosens the latch keeping the ladder extended and the ladder begins to shorten at a rate of 0.5 m/s. The top of the ladder maintains contact with the wall throughout, but begins to slide down the wall. How fast is the top of the ladder sliding down the wall after 2 s?

[Total = 100]

Part Z. Bonus problems! If you feel like it and have the time, do one or both of these.

0. A standard 8 × 8 chessboard has two opposite corner squares removed. Each of a set of dominos is a rectangle that is exactly the size of two adjacent squares of the board. Show how to lay such dominos down on the board such that each domino covers two adjacent squares of the mutilated board and so that no part of the board is left uncovered, or show why it is impossible to cover the board with dominos in such a way. *[1]*



00. Write a haiku touching on calculus or mathematics in general. *[1]*

What is a haiku?

seventeen in three:
five and seven and five of
syllables in lines

ENJOY THE BREAK!

(This exam brought to you by Trent University, the Department of Mathematics, and their minions.)