Mathematics 1110H – Calculus I: Limits, derivatives, and Integrals
TRENT UNIVERSITY, Fall 2019

Assignment #4
Not the Zero Function
Due on Wednesday, 6 November.

The following function was used as an example by Augustin-Louis Cauchy when investigating the convergence of Taylor series.

$$f(x) = \begin{cases} e^{-1/x^2} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

1. Verify that $f(x)$ is continuous at $x = 0$. *[4]*

2. Show that $f'(0)$ is defined and equal to 0. *[6]*

NOTE. It turns out that the second, third, fourth – every! – derivative of $f(x)$ is defined and equal to 0 at $x = 0$, making it indistinguishable from the zero function, $g(x) = 0$ for all x, as far as far as calculus can determine it from its behavious at $x = 0$.