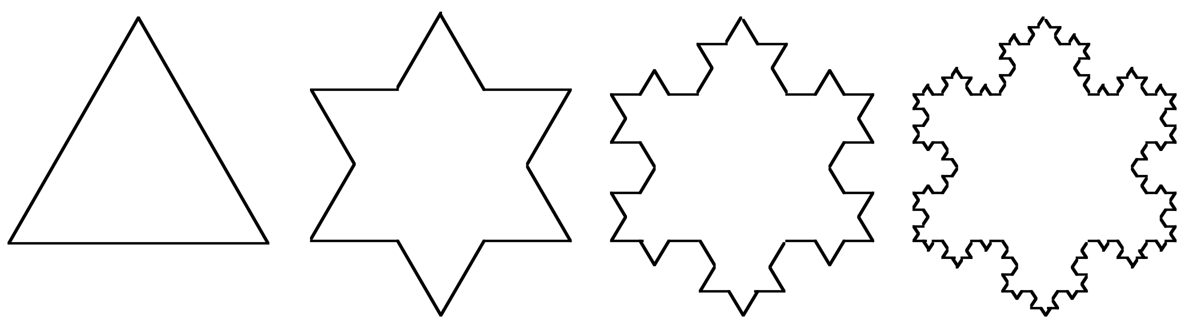
Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Fall 2018

Assignment #6
The ultimate snowflake - because winter is coming!
Due on Friday, 2 November.

Suppose one starts with an equilateral triangle with sides of length 1. If one modifies each of the line segments composing the triangle by cutting out the middle third of the segment, and then inserting an outward-pointing "tooth," both of whose sides are as long as the removed third, one gets a six-pointed star. Repeating this for each of the line segments making up the star, then to each of the line segments making up the resulting figure, and so on, leads gives increasingly intricate curves with more and more corners. The first stages of the process are pictured in the diagram below:



Note that the lengths of the line segments at each stage are a third of the length of the segments at the preceding stage. For the sake of being definite, let's say we have the triangle at step 0 of the process, the six-pointed star at step 1 of the process, the next shape at step 2 of the process, and so on. The curve which is the limit of this process, if one takes infinitely many steps, is often called the *snowflake curve**. We will investigate the length of this curve and the area of the region that it encloses.

1. Find the length of the snowflake curve. *[5]*

2. Find the area of the region enclosed by the snowflake curve. *[5]*

* Also known as the *Koch curve*. Note that "curve" does not imply smoothness here: although it is continuous, the snowflake curve has infinitely many corners and is therefore none too differentiable ...