Mathematics 1101Y – Calculus I: functions and calculus of one variable
TRENT UNIVERSITY, 2013–2014
Test #2
Monday, 13 February, 2014.
Time: 50 minutes

Instructions

- *Show all your work.* Legibly, please!
- *If you have a question, ask it!*
- Use page 4 and the back sides of all the pages for rough work or extra space. (There is also a bonus question on the bottom of page 4.)
- You may use a calculator and an aid sheet.

1. Compute any *four* (4) of the integrals in parts **a**–**f**. *[16 = 4 × 4 each]*

a. $\displaystyle\int \frac{1}{u\ln(u)}\,du$ **b.** $\displaystyle\int_{-12}^{-9} \frac{1}{\sqrt{v+13}}\,dv$ **c.** $\displaystyle\int w\sec^2(w)\,dw$

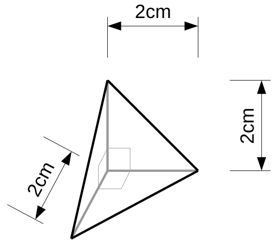
d. $\displaystyle\int_0^1 \left(x^2+1\right)^2\,dx$ **e.** $\displaystyle\int \sec^4(y)\,dy$ **f.** $\displaystyle\int_0^{1/2} \cos(2\pi z)\,dz$

2. Do any *two* (2) of parts **g**–**k**. *[14 = 2 × 7 each]*

g. The volume of a solid obtained by revolving a region about the y-axis is given by $V = \displaystyle\int_0^{\pi/4} \pi\tan^2(y)\,dy$ when computed using the disk method. Sketch this solid and find its volume.

h. Find the average value of $f(x) = \cos(x) + \sin(x)$ on the interval $[0, \pi]$.

i. Compute $\displaystyle\int_0^2 x^2\,dx$ using the Left- or the Right-hand Rule.



j. A corner of a cube with sides 2 *cm* long is cut off along a plane passing through the three neighbouring vertices. Find the volume of this solid. [Picture at right! ⟹]

k. Sketch the region enclosed by $y = x^2$ and $x = y^2$ and find its area.

3. Do *one* (1) of parts **l** or **m**. Both involve the region between $y = \sin(x)$ and $y = 0$, where $0 \le x \le \pi$.

l. Sketch the solid of revolution obtained by revolving the given region about the y-axis and find its volume using the cylindrical shell method. *[10]*

m. Sketch the solid of revolution obtained by revolving the given region about the x-axis and find its volume using the disk/washer method. *[10]*

[Total = 40]

Bonus. Suppose a number of circles are drawn on a piece of paper, dividing it up into regions whose borders are made up of circular arcs. Show that one can always colour these regions using only black and white so that no two regions that have a border arc in common have the same colour. *[1]*