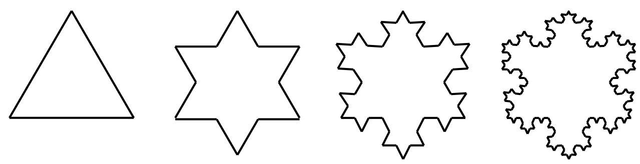
Mathematics 1101Y – Calculus I: Functions and calculus of one variable
TRENT UNIVERSITY, 2013–2014

Assignment #6
Snow Pies?
Due on Monday, 31 March, 2014.

Suppose one has an equilateral triangle with sides of length 1. If one modifies each of the line segments composing the triangle by cutting out the middle third of the segment, and then inserting an outward-pointing "tooth," both of whose sides are as long as the removed third, one gets a six-pointed star. Suppose one repeats this process for each of the line segments making up the star, then to each of the line segments making up the resulting figure, and so on, as in the diagram:



Note that the lengths of the line segments at each stage are a third of the length of the segments at the preceding stage. The curve which is the limit of this process, after infinitely many steps, is often called the *snowflake* or *Koch* curve.

1. Find the length of the snowflake curve. *[2]*

2. Find the area of the region enclosed by the snowflake curve. *[2]*

Recall that if $|r| < 1$, then the geometric series with common ratio r and initial term a has a nice sum:

$$a + ar + ar^2 + ar^3 + \cdots = \sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

A *power series*, which we haven't really gotten to in class yet, can be thought of as an infinite degree polynomial,

$$a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots = \sum_{n=0}^{\infty} a_nx^n\,,$$

where, of course, each coefficient a_n is a real number. It is a fact, sometimes called *Abel's Lemma* (which we will *not* prove in this course), that if $\sum_{n=0}^{\infty} a_n$ converges, then $\sum_{n=0}^{\infty} a_n = \lim_{x \to 1^-} \left(\sum_{n=0}^{\infty} a_nx^n \right)$.

3. Use the sum of a geometric series formula in reverse to write $f(x) = \dfrac{1}{1+x^2}$ as a power series. For which values of x does this work? *[1]*

4. Use the series from **3** and the fact that $\arctan(x) = \displaystyle\int_0^x \frac{1}{1+t^2}\,dt$ to write $\arctan(x)$ as a power series. For which values of x should this work? (If there is any justice ... :-) *[2]*

5. Use the series from **4** and Abel's Lemma to show that $\dfrac{\pi}{4} = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} = 1 - \frac{1}{3} + \frac{1}{5} - \cdots$. *[2]*

6. How many terms of Gregory's series do you need to guarantee that the corresponding partial sum is within $0.01 = \frac{1}{100}$ of π? *[1]*