

Mathematics 1101Y – Calculus I: Functions and calculus of one variable  
TRENT UNIVERSITY, 2011–2012

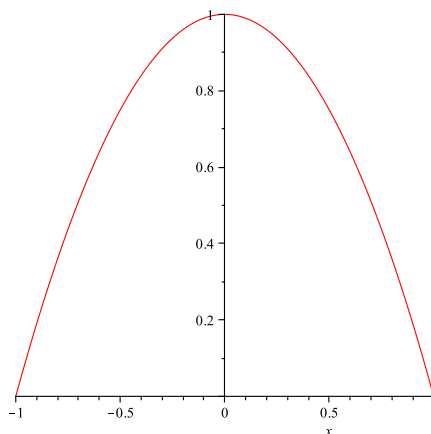
Solutions to Assignment #1  
Plotting with Maple<sup>†</sup>

The solutions below are given in terms of “classic worksheet”-style Maple commands, since these are much easier to describe than their graphical user interface equivalents.

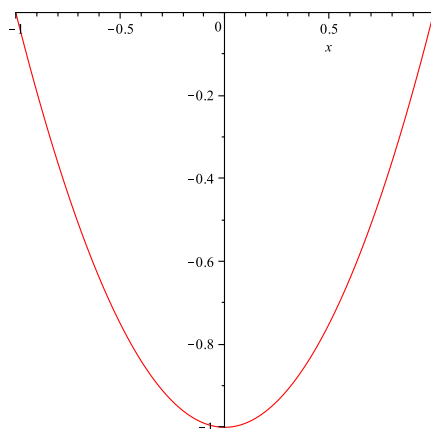
1. Use Maple to plot the curves defined by  $y = 1 - x^2$ ,  $y = x^2 - 1$ ,  $y = \sqrt{1 - x}$ , and  $y = -\sqrt{1 - x}$ , respectively, for  $-1 \leq x \leq 1$  in each case. [Please submit a printout of your worksheet(s).] [2]

SOLUTION. Suitable instances of the `plot` command and their output are given below. [The graphs have been reduced in size to save some space.]

```
> plot(1-x^2,x=-1..1);
```



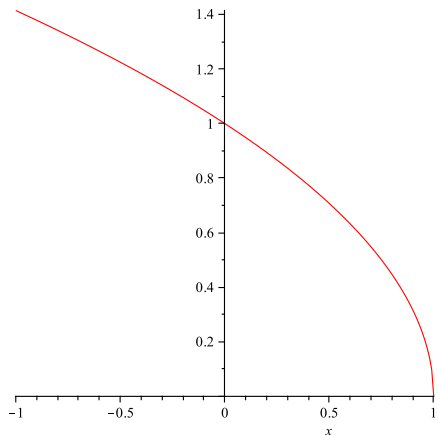
```
> plot(x^2-1,x=-1..1);
```



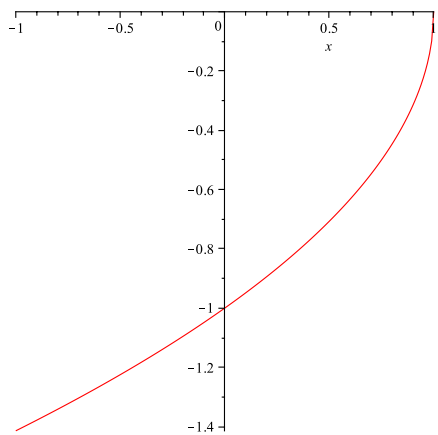
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<sup>†</sup> ... though you may sometimes feel as if Maple is plotting *against* you.

```
> plot(sqrt(1-x),x=-1..1);
```



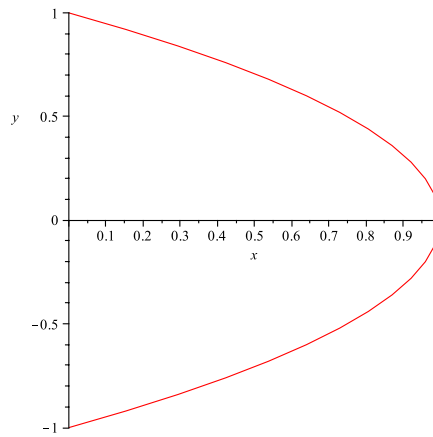
```
> plot(-sqrt(1-x),x=-1..1);
```



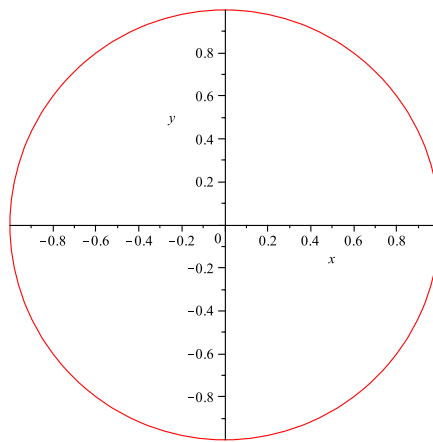
2. Use Maple to plot the curves implicitly defined by  $x = 1 - y^2$ , for  $-1 \leq y \leq 1$ ,  $x^2 + y^2 = 1$ ,  $(x^2 + y^2)^2 = 2(x^2 - y^2)$ , and  $(x^2 + y^2)^3 = 4x^2y^2$ , respectively, the latter three for all  $x$  and  $y$  satisfying each equation. [Please submit a printout of your worksheet(s).] [2]

SOLUTION. The `plots` package in Maple provides additional commands for various specialized purposes. Suitable instances of the `implicitplot` command from this package and their output are given below. [The graphs have been reduced in size to save some space.]

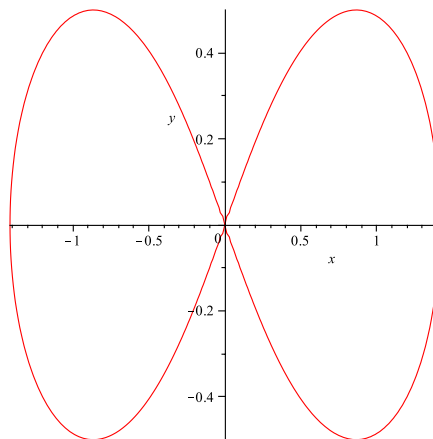
```
> with(plots,implicitplot)
> implicitplot(x=1-y^2,x=-1..1,y=-1..1);
```



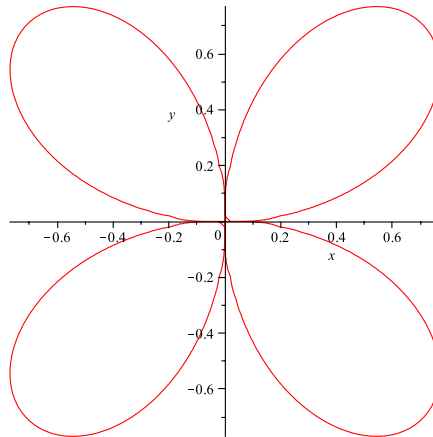
```
> implicitplot(x^2+y^2=1,x=-1..1,y=-1..1);
```



```
> implicitplot((x^2+y^2)^2=2*(x^2-y^2),x=-1.5..1.5,y=-1..1,gridrefine=2);
```



```
> implicitplot(4*x^2*y^2=(x^2+y^2)^3,x=-1..1,y=-1..1,gridrefine=2);
```

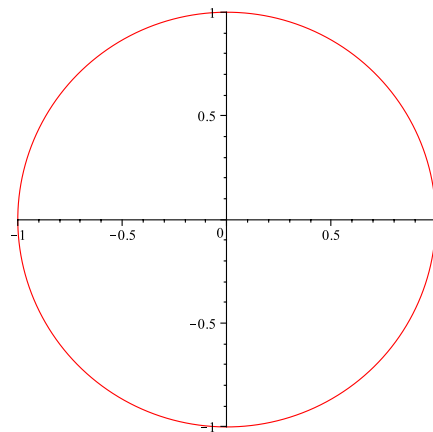


The limits for  $x$  and  $y$  in the last two graphs were tinkered with until the entire curve being graphed was visible in the output. Note also the use of the option `gridrefine=2` in these two graphs as well; this option tries to make the output as smooth as possible and each of the two graphs would be graphed rather less accurately without it. ■

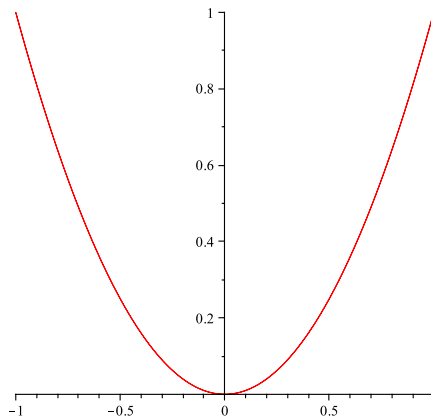
3. Use **Maple** to plot the parametric curves given by  $x = \cos(t)$  and  $y = \sin(t)$ ,  $x = \sin(t)$  and  $y = \frac{1}{2} - \frac{1}{2} \cos(2t)$ , and  $x = \sec(t)$  and  $y = \tan^2(t)$ , respectively, for  $0 \leq t \leq 2\pi$  in each case. [Please submit a printout of your worksheet(s).] [1.5]

SOLUTION. Suitable instances of the `plot` command, which can handle parametric curves, and their output are given below. [The graphs have been reduced in size to save some space.]

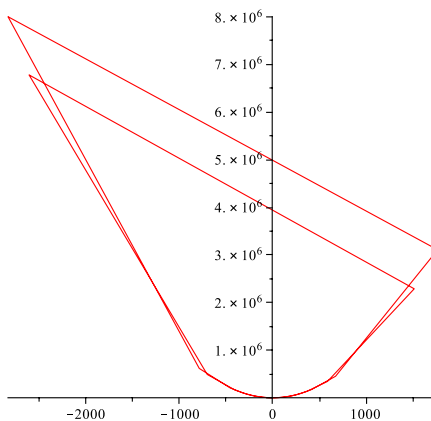
```
> plot([cos(t),sin(t),t=0..2*Pi]);
```



```
> plot([sin(t),1/2-(1/2)*cos(2*t),t=0..2*Pi]);
```



```
> plot([sec(t),tan(t)^2,t=0..2*Pi]);
```

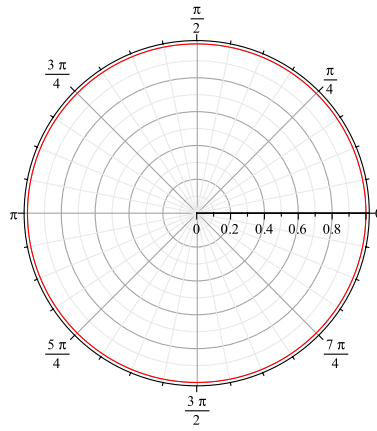


If you think the last graph just isn't quite right, *you* are quite right. Part of the problem is that both  $\sec(t)$  and  $\tan(t)$  are seriously discontinuous when  $\cos(t) = 0$  which happens twice for  $0 \leq t \leq 2\pi$ . Maple does its best, but the result is hard to interpret. ■

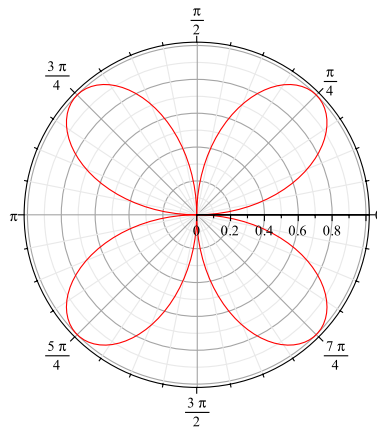
4. Use Maple to plot the curves in polar coordinates given by  $r = 1$ ,  $r = \sin(2\theta)$ , and  $r = \sqrt{2} \cos(\theta)$ , respectively, for  $0 \leq \theta < 2\pi$  in each case. [Please submit a printout of your worksheet(s).] [1.5]

SOLUTION. As noted in the solution to 2, the `plots` package in Maple provides additional commands for various specialized purposes. Suitable instances of the `polarplot` command from this package and their output are given below. [The graphs have been reduced in size to save some space.]

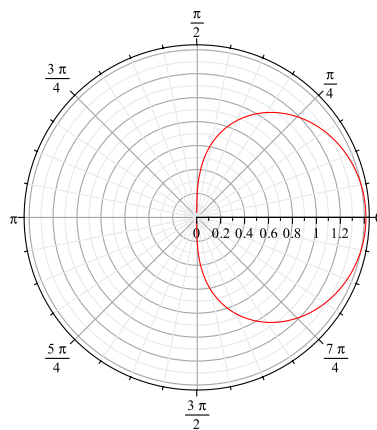
```
> polarplot(1,theta=0..2*Pi);
```



```
> polarplot(sin(2*theta),theta=0..2*Pi);
```



```
> polarplot(sqrt(2*cos(theta)),theta=0..2*Pi);
```



5. Some of the curves in problems 1–4 are actually the same curve. (With different presentations ... ) Which ones are the same? [2]

SOLUTION. *i.* The curves given by  $x^2 + y^2 = 1$  in **2**, parametrically by  $x = \cos(t)$  and  $y = \sin(t)$  for  $0 \leq t \leq 2\pi$  in **3**, and by the polar equation  $r = 1$ ,  $0 \leq \theta \leq 2\pi$ , in **4** are all the same: this is just the circle with radius 1 centred at the origin.

*ii.* The curves given by  $(x^2 + y^2)^3 = 4x^2y^2$  in **2** and by the polar equation  $r = \sin(2\theta)$ ,  $0 \leq \theta \leq 2\pi$ , in **4** are the same too. (Hey - four-leaf clovers are not that common! :-)

There are also some near misses:

*iii.* The curves  $y = \sqrt{1-x}$  and  $y = -\sqrt{1-x}$ , where  $-1 \leq x \leq 1$ , in **1** are the two halves of the (piece of the) parabola  $x = 1 - y^2$ ,  $-1 \leq y \leq 1$ , in **2**.

*iv.* The curve  $y = x^2 - 1$ ,  $-1 \leq x \leq 1$ , in **1** and the parametric curve given by  $x = \sec(t)$  and  $y = \tan^2(t)$ , for  $0 \leq t \leq 2\pi$ , (at least those  $t$  for which it is defined) in **3** are both parts of the parabola  $y = x^2 - 1$ .

*v.* Several of the curves – you figure out just which! – are parts of parabolas, all of which have the same shape, but differ in position and/or orientation. ■

6. Pick one of the curves with different presentations that are the same and explain, in detail, why the different presentations give the same points. [1]

SOLUTION. This is pretty easy to do for the unit circle.

First, if  $x = \cos(t)$  and  $y = \sin(t)$  for some  $t \in [0, 2\pi]$ , then  $x^2 + y^2 = \cos^2(t) + \sin^2(t) = 1$ , so every point on the parametric curve is on the unit circle.

Second, if  $x^2 + y^2 = 1$  for some point  $(x, y)$ , then  $r = \sqrt{x^2 + y^2} = \sqrt{1} = 1$ , so every point on the unit circle is on the polar curve.

Third, if  $(r, \theta)$  is on the polar curve  $r = 1$  for some  $0 \leq \theta \leq 2\pi$ , then for  $t = \theta$  we have  $x = r \cos(\theta) = 1 \cos(t) = \cos(t)$  and  $y = r \sin(\theta) = 1 \sin(t) = \sin(t)$ , so every point on the polar curve is on the parametric curve.

It follows that every point on each curve is also on the other curves, so they are really the same set of points, *i.e.* they are all the same curve. ■