MATH 1101Y 2009 Quiz 19 (b)

1. Determine whether the series is convergent or divergent.

   (a) (2 pts)

$$\sum_{k=2}^{\infty} \frac{1}{n\,(\ln n)}$$

Solution: We use the Integral Test.

$$\begin{aligned}
& \int_2^{\infty} \frac{1}{x \ln x} dx \\
= & \lim_{b\to\infty} \int_2^{b} \frac{1}{x \ln x} dx \\
\text{(Let } u \text{ be } \ln x.) \quad = & \lim_{b\to\infty} \int_{\ln 2}^{\ln b} \frac{1}{u} du \\
= & \lim_{b\to\infty} [\ln u]_{\ln 2}^{\ln b} = \lim_{b\to\infty} (\ln\ln b - \ln\ln 2) = \infty.
\end{aligned}$$

Therefore, the series is divergent. □

   (b) (1 pts)

$$\sum_{n=1}^{\infty} \frac{3n^2 - 2n + 3}{2n^3 + n - 1}$$

Solution: We apply the limit form of the Comparison Test. We compare this series to the series

$$\sum_{n=1}^{\infty} \frac{n^2}{n^3} = \sum_{n=1}^{\infty} \frac{1}{n}.$$

Since

$$\begin{aligned}
\lim_{n\to\infty} \frac{\frac{3n^2-2n+3}{2n^3+n-1}}{\frac{n^2}{n^3}} &= \lim_{n\to\infty} \frac{3n^2 - 2n + 3}{2n^3 + n - 1} \cdot \frac{n^3}{n^2} \\
&= \frac{3}{2},
\end{aligned}$$

and $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent, this series is divergent. □

   (c) (2 pts)

$$\sum_{n=2}^{\infty} (-1)^{n-1} \frac{n}{e^{\frac{1}{n}}}$$

Solution: We apply the Divergence Test.

$$\lim_{n\to\infty} \frac{n}{e^{\frac{1}{n}}} = \frac{\infty}{1} = \infty \neq 0,$$

this series is divergent. □