

MATH 1101Y 2009 Quiz 16 (b)

1. (2 pts) Eliminate the parameter to find a Cartesian equation of the curve.

$$x = t^2, \quad y = \sin 2t$$

Solution: Since $t = \sqrt{x}$, the Cartesian equation of the curve is

$$y = \sin(2\sqrt{x}).$$

□

2. (3 pts) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the equation

$$x = t^2 + t^4, \quad y = t + t^3.$$

Solution:

$$\frac{dy}{dt} = 1 + 3t^2, \quad \frac{dx}{dt} = 2t + 4t^3.$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1 + 3t^2}{2t + 4t^3}.$$

$$\frac{d}{dt} \left(\frac{dy}{dx} \right) = \frac{6t(2t + 4t^3) - (1 + 3t^2)(2 + 12t^2)}{(2t + 4t^3)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}} = \frac{6t(2t + 4t^3) - (1 + 3t^2)(2 + 12t^2)}{(2t + 4t^3)^3}.$$

□