Mathematics 110 – Calculus of one variable
Trent University 2002-2003

§A Quizzes

Quiz #1. Wednesday, 18 September, 2002. *[10 minutes]*

12:00 Seminar

1. Compute $\displaystyle\lim_{x\to 2}\frac{x^2-x-2}{x-2}$ or show that this limit does not exist. *[5]*
2. Sketch the graph of a function $f(x)$ which is defined for all x and for which $\displaystyle\lim_{x\to 0} f(x) = 1$, $\displaystyle\lim_{x\to 2^+} f(x)$ does not exist, and $\displaystyle\lim_{x\to 2^-} f(x) = 4$. *[5]*

13:00 Seminar

1. Compute $\displaystyle\lim_{x\to 2^-}\frac{x^2-x+2}{x-2}$ or show that this limit does not exist. *[5]*
2. Sketch the graph of a function $g(x)$ which is defined for all x, and for which $\displaystyle\lim_{x\to 0} g(x) = \infty$, $\displaystyle\lim_{x\to 2} g(x)$ does not exist, and $g(x)$ does not have an asymptote at $x = 2$. *[5]*

Quiz #2. Wednesday, 25 September, 2002. *[10 minutes]*

12:00 Seminar

1. Use the $\epsilon - \delta$ definition of limits to verify that $\displaystyle\lim_{x\to 3}(5x-7) = 8$. *[10]*

13:00 Seminar

1. Use the $\epsilon - \delta$ definition of limits to verify that $\displaystyle\lim_{x\to 2}(3-2x) = -1$. *[10]*

Quiz #3. Wednesday, 2 October, 2002. *[10 minutes]*

12:00 Seminar

1. For which values of the constant c is the function

$$f(x) = \begin{cases} e^{cx} & x \geq 0 \\ cx+1 & x < 0 \end{cases}$$

continuous at $x = 0$? Why? *[10]*

13:00 Seminar

1. For which values of the constant c is the function

$$f(x) = \begin{cases} e^{cx} & x \geq 0 \\ c(x+1) & x < 0 \end{cases}$$

continuous at $x = 0$? Why? *[10]*

Quiz #4. Wednesday, 9 October, 2002. *[12 minutes]*

12:00 Seminar

Suppose

$$f(x) = \begin{cases} x & x < 0 \\ 0 & x = 0 \\ 2x^2 + x & x > 0 \end{cases} .$$

1. Use the definition of the derivative to check whether $f'(0)$ exists and compute it if it does. *[7]*
2. Compute $f'(1)$ (any way you like). *[3]*

13:00 Seminar

Suppose $g(x) = \dfrac{1}{x+1}$. Compute $g'(x)$ using

1. the rules for computing derivatives *[3]*, and
2. the definition of the derivative. *[7]*

Quiz #5. Wednesday, 16 October, 2002. *[10 minutes]*

12:00 Seminar

Compute $\dfrac{d}{dx}\sqrt[5]{x}$ using

1. the Power Rule *[2]*, and
2. the fact that $f(x) = \sqrt[5]{x}$ is the inverse function of $g(x) = x^5$. *[8]*

13:00 Seminar

1. Compute $\dfrac{d}{dx}\arccos(x)$ given that $\cos(\arccos(x)) = x$ and $\cos^2(x) + \sin^2(x) = 1$. *[10]*

Quiz #6. Wednesday, 30 October, 2002. *[10 minutes]*

12:00 Seminar

1. Find the absolute and local maxima and minima of $f(x) = x^3 + 2x^2 - x - 2$ on $[-2, 2]$. *[10]*

13:00 Seminar

1. Find the absolute and local maxima and minima of $f(x) = x^3 - 3x^2 - x + 3$ on $[-2, 2]$. *[10]*

Quiz #7. Wednesday, 6 November, 2002. *[15 minutes]*

12:00 Seminar

1. Find the intercepts, critical and inflection points, and horizontal asymptotes of $f(x) = (x-2)e^x$ and sketch its graph. *[10]*

13:00 Seminar

1. Find the intercepts, critical and inflection points, and horizontal asymptotes of $h(x) = (x+1)e^{-x}$ and sketch its graph. *[10]*

Quiz #8. Wednesday, 27 November, 2002. *[15 minutes]*

12:00 Seminar

1. Compute:

$$\int_1^{e^\pi} \frac{1}{x} \sin\left(\ln(x)\right)\, dx \qquad [5]$$

2. What definite integral does the Right-hand Rule limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{i}{n}\right) \cdot \frac{1}{n}$$

correspond to? *[5]*

13:00 Seminar

1. Compute:

$$\int_0^{\pi/4} \frac{\tan(x)}{\cos^2(x)}\, dx \qquad [5]$$

2. What definite integral does the Right-hand Rule limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n} - 1\right) \cdot \frac{1}{n}$$

correspond to? *[5]*

Quiz #9. Wednesday, 4 December, 2002. *[15 minutes]*

12:00 Seminar

1. Find the area of the region enclosed by $y = -x^2$ and $y = x^2 - 2x$. *[10]*

13:00 Seminar

1. Find the area of the region enclosed by $y = (x-2)^2 + 1 = x^2 - 4x + 5$ and $y = x + 1$. *[10]*

Quiz #10. Wednesday, 8 January, 2003. *[25 minutes]*

12:00 Seminar

1. Sketch the solid obtained by rotating the region bounded by $y = 0$ and $y = \cos(x)$ for $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$ about the y-axis and find its volume. *[10]*

13:00 Seminar

1. Sketch the solid obtained by rotating the region bounded by $y = -1$ and $y = \cos(x)$ for $0 \le x \le \pi$ about the y-axis and find its volume. *[10]*

Quiz #11. Wednesday, 15 January, 2003. *[20 minutes]*

12:00 Seminar

1. Compute $\displaystyle\int \frac{1}{1 - x^2}\, dx$. *[10]*

13:00 Seminar

1. Compute $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}}\,dx$. *[10]*

Quiz #12. Wednesday, 22 January, 2003. *[20 minutes]*

12:00 Seminar

1. Compute $\displaystyle\int \frac{3x^2+4x+2}{x^3+2x^2+2x}\,dx$. *[10]*

13:00 Seminar

1. Compute $\displaystyle\int \frac{2x+1}{x^3+2x^2+x}\,dx$. *[10]*

Quiz #13. Wednesday, 29 January, 2003. *[15 minutes]*

12:00 Seminar

1. Compute $\displaystyle\int_{-\infty}^{\infty} e^{-|x|}\,dx$ or show that it does not converge. *[10]*

13:00 Seminar

1. Compute $\displaystyle\int_{-1}^{1} \frac{x+1}{\sqrt[3]{x}}\,dx$ or show that it does not converge. *[10]*

Quiz #14. Wednesday, 5 February, 2003. *[20 minutes]*

12:00 Seminar

1. Sketch the solid obtained by rotating the region bounded by $x=0$, $y=4$ and $y=x^2$ for $0 \le x \le 2$ about the y-axis. *[2]*
2. Compute the surface area of this solid. *[8]*

13:00 Seminar

1. Sketch the curve given by the parametric equations $x = 1+\cos(t)$ and $y=\sin(t)$, where $0 \le t \le 2\pi$. *[3]*
2. Compute the arc-length of this curve using a suitable integral. *[7]*

Quiz #15. Wednesday, 26 February, 2003. *[20 minutes]*

12:00 Seminar

1. Graph the polar curve $r = \sin(2\theta)$, $0 \le \theta \le 2\pi$. *[4]*
2. Find the area of the region enclosed by this curve. *[6]*

13:00 Seminar

1. Graph the polar curve $r=\cos(\theta)$, $0 \le \theta \le 2\pi$. *[4]*
2. Find the arc-length of this curve. *[6]*

Quiz #16. Wednesday, 5 March, 2003. *[15 minutes]*

12:00 Seminar

Let $a_k = \dfrac{1}{(k+1)(k+2)}$ and $s_n = \displaystyle\sum_{k=0}^{n} a_k$.

1. Find a formula for s_n in terms of n. *[5]*
2. Does $\displaystyle\sum_{k=0}^{\infty} a_k$ converge? If so, what does it converge to? *[5]*

13:00 Seminar

Let $a_k = \ln\left(\dfrac{k}{k+1}\right)$ and $s_n = \displaystyle\sum_{k=0}^{n} a_k$.

1. Find a formula for s_n in terms of n. *[5]*
2. Does $\displaystyle\sum_{k=0}^{\infty} a_k$ converge? If so, what does it converge to? *[5]*

Quiz #17. Wednesday, 12 March, 2003. *[15 minutes]*

12:00 Seminar

Determine whether each of the following series converges or diverges:

1. $\displaystyle\sum_{n=0}^{\infty} e^{-n}$ *[5]* 2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\arctan(n)}$ *[5]*

13:00 Seminar

Determine whether each of the following series converges or diverges:

1. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n+1}$ *[5]* 2. $\displaystyle\sum_{n=1}^{\infty} 2^{1/n^2}$ *[5]*

Quiz #18. Wednesday, 19 March, 2003. *[15 minutes]*

12:00 Seminar

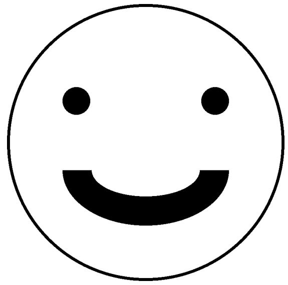
Determine whether each of the following series converges absolutely, converges conditionally, or diverges:

1. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n^2+2}$ *[5]* 2. $\displaystyle\sum_{n=1}^{\infty} \frac{n!(-1)^n}{n^n}$ *[5]*

13:00 Seminar

Determine whether each of the following series converges absolutely, converges conditionally, or diverges:

1. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \left(2n^2+3n+4\right)}{3n^2+4n+5}$ *[5]* 2. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^2}$ *[5]*



Bonus Quiz. Friday, 21 March, 2003. *[15 minutes]*

1. A smiley face is drawn on the surface of a balloon which is being inflated at a rate of $10\ cm^3/s$. At the instant that the radius of the balloon is $10\ cm$ the eyes are $10\ cm$ apart, as measured *inside* the balloon. How is the distance between them changing at this moment? *[10]*

Quiz #19. Wednesday, 26 March, 2003. *[20 minutes]*

12:00 Seminar

Consider the power series $\displaystyle\sum_{n=0}^{\infty} \frac{2^n x^{2n}}{n!}$.

1. For which values of x does this series converge? *[6]*
2. This series is equal to a (reasonably nice) function. What is it? Why? *[4]*

13:00 Seminar

Consider the power series $\displaystyle\sum_{n=0}^{\infty} \frac{2^n x^{n+1}}{n+1}$.

1. For which values of x does this series converge? *[6]*
2. This series is equal to a (reasonably nice) function. What is it? Why? *[4]*

Quiz #20. Wednesday, 2 April, 2003. *[20 minutes]*

12:00 Seminar

Let $f(x) = \sin(\pi - 2x)$.

1. Find the Taylor series at $a = 0$ of $f(x)$. *[6]*
2. Find the radius and interval of convergence of this Taylor series. *[4]*

13:00 Seminar

Let $f(x) = \ln(2 + x)$.

1. Find the Taylor series at $a = 0$ of $f(x)$. *[6]*
2. Find the radius and interval of convergence of this Taylor series. *[4]*