Linear Functions

$y = y_1 + m(x - x_1)$

$y = mx + b$

Line through $(0, b)$ slope m.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Quadratic Functions

$y = a(x-h)^2 + k$

Parabola opens up if $a > 0$

vertex at (h, k)

$y = ax^2 + bx + c$

Parabola opens up if $a > 0$

vertex at $(-\frac{b}{2a}, f(-\frac{b}{2a}))$

$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Circles

$(x-h)^2 + (y-k)^2 = r^2$

radius r, centre (h, k)

Ellipse

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

radius r and centre (h, k)

vertices a units right/left from centre and b units up/down from centre

Completing the square

Divide by coefficient of x^2

Move constant to other side

Take half of coefficient of x

Square it, add to both sides

Factor the left side (root), solve for x.

fractions

$ab + ac = a(b+c)$

$\frac{(\frac{a}{b})}{c} = \frac{a}{bc}$

$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

$\frac{a-b}{c-d} = \frac{b-a}{d-c}$

$\frac{ab+ac}{a} = b + c,\ a \neq 0$

$a\left(\frac{b}{c}\right) = \frac{ab}{c}$

$\frac{a}{(\frac{b}{c})} = \frac{ac}{b}$

$\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$

$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$\frac{(\frac{a}{b})}{(\frac{c}{d})} = \frac{ad}{bc}$

Radicals

$\sqrt[n]{a} = a^{\frac{1}{n}}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a}$

$\sqrt[n]{a^n} = a$, if n is odd

$\sqrt[n]{a^n} = |a|$, if n is even

$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Logarithms

$y = \log_b x \equiv x = b^y$

$\log_b b = 1$

$\log_b 1 = 0$

$\log_b b^x = x$

$b^{\log_b x} = x$

$\log_b(x^r) = r\log_b x$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b(\frac{x}{y}) = \log_b x - \log_b y$

Factoring

$x^2 - a^2 = (x+a)(x-a)$

$x^2 + 2ax + a^2 = (x+a)^2$

$x^2 - 2ax + a^2 = (x-a)^2$

$x^2 + (a+b)x + ab$
$= (x+a)(x+b)$

$x^3 + 3ax^2 + 3a^2x + a^3$
$= (x+a)^3$

$x^3 - 3ax^2 + 3a^2x - a^3$
$= (x-a)^3$

$x^3 + a^3$
$= (x+a)(x^2 - ax + a^2)$

$x^3 - a^3$
$= (x-a)(x^2 + ax + a^2)$

Pythagorean identities

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

Reciprocal functions

$\cot x = \frac{1}{\tan x}$

$\csc x = \frac{1}{\sin x}$

$\sec x = \frac{1}{\cos x}$

Co-function identities

$\sin(\frac{\pi}{2} - x) = \cos x$

$\cos(\frac{\pi}{2} - x) = \sin x$

$\tan(\frac{\pi}{2} - x) = \cot x$

$\cot(\frac{\pi}{2} - x) = \tan x$

$\sec(\frac{\pi}{2} - x) = \csc x$

$\csc(\frac{\pi}{2} - x) = \sec x$

Double angles

$\sin(2x) = 2\sin x\cos x$

$\cos(2x) = \cos^2 x - \sin^2 x$
$= 2\cos^2 x - 1$
$= 1 - 2\sin^2 x$

$\tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$

Even/odd

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\tan(-x) = -\tan x$

Half angles

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$
$= \frac{\sin x}{1+\cos x}$

Power reducing formulas

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\tan^2 x = \frac{1-\cos 2x}{1+\cos 2x}$

Limits

$\lim_{x\to\infty} e^x = \infty$

$\lim_{x\to-\infty} e^x = 0$

$\lim_{x\to\infty} \ln x = \infty$

$\lim_{x\to 0+} \ln x = -\infty$

If $r > 0$, $\lim_{x\to\infty} \frac{b}{x^r} = 0$

If $r > 0$ and x^r is real for $-x$,

$\lim_{x\to-\infty} \frac{b}{x^r} = 0$

n is even $\lim_{x\to\pm\infty} x^n = \infty$

n is odd $\lim_{x\to\infty} x^n = \infty$

$\lim_{x\to-\infty} x^n = -\infty$

L'Hôpital's rule

If $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0}$

or $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\pm\infty}{\pm\infty}$

then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

Harmonic Series:

If $a_n = \frac{1}{n}$

$\sum_{k=1}^{\infty} a_k = \frac{1}{1} + \cdots + \frac{1}{n}$

Divergent to $+\infty$

Quadratic Series:

If $a_n = \frac{1}{n^2}$

$\sum_{k=1}^{\infty} a_k = \frac{1}{1^2} + \cdots + \frac{1}{n^2}$

Convergent to $\frac{\pi^2}{6}$

Exponential Series:

If $a_n = \frac{1}{n!}$

$\sum_{k=1}^{\infty} a_k = \frac{1}{0!} + \cdots + \frac{1}{n!}$

Convergent to e

Common Taylor series

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$
$= 1 + x + x^2 + \cdots$

for $-1 < x < 1$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
$= 1 + x + \frac{x^2}{2!} + \cdots$

$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$
$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$

$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$
$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$

Note the similarity between $\sin x$, $\cos x$, and e^x.

How to Find a Taylor series

Given a smooth function f, we can always write down a Taylor series; there is no guarantee that the series converges to anything, let alone to the function.

Given a smooth function $f : \mathbb{R} \to \mathbb{R}$, its Taylor series (around 0) is

$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)x^n}{n!}$

A common mistake is to use $f^{(n)}(x)$ instead of $f^{(n)}(0)$.

Given a smooth function $f : \mathbb{R} \to \mathbb{R}$, its Taylor series expanded around a is

$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}$

The most important example of this sort of Taylor series is $\log x$

$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n}$

$= (x-1) - \frac{(x-1)^2}{2}$
$+ \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4}$
$+ \cdots$

Convergence

An infinite series

$\sum_{i=1}^{\infty} = a_1 + a_2 + \cdots + a_n$

converges to L

if the sequence of partial sums

$s_n = a_1 + a_2 + \cdots a_n$

converges to a limit L.

This definition says that as we add the terms in the infinite string above, the answer gets closer and closer to L, and does not "jump around".

Zero test

If the series $\sum_{i=1}^{\infty} a_i$ converges, then the terms $a_i \to 0$.

The test says that if the terms a_i do not go to zero, *then there is **no way*** for the series of partial sums to converge.

Done. Does NOT converge.

Integral Test

Let $a_i = f(i)$, where $f(x)$ is a continuous function with $f(x) > 0$, and is decreasing.

Then the series $\sum_{i=1}^{\infty} a_i$ converges if the improper integral $\int_1^{\infty} f(x)dx < \infty$.

The series $\sum_{i=1}^{\infty} a_i$ diverges if the improper integral $\int_1^{\infty} f(x)dx = \infty$.

One application is the convergence of the "p-series":

$\sum_{n=1}^{\infty} \frac{1}{n^p}$

converges if $p > 1$, and diverges if $p \leq 1$

Comparison Test

Suppose that $\sum_1^{\infty} a_i$ and $\sum_1^{\infty} b_i$ are series with all terms positive so $a_i \geq 0$ and $b_i \geq 0$.

$\sum_{i=1}^{\infty} b_i$ is convergent and $a_i \leq b_i$ for all i

$\Longrightarrow \sum_{i=1}^{\infty} a_i$ is convergent.

$\sum_{i=1}^{\infty} b_i$ is divergent and $a_i \geq b_i$ for all i

$\Longrightarrow \sum_{i=1}^{\infty} a_i$ is divergent.

Limit Comparison Test

Suppose that $\sum_1^{\infty} a_i$ and $\sum_1^{\infty} b_i$ are series with all terms positive.

$\lim_{i\to\infty} \frac{a_i}{b_i} = c > 0$

$\Longrightarrow \sum_{i=1}^{\infty} a_i$ and $\sum_{i=1}^{\infty} b_i$ either both converge, or both diverge.

$\lim_{i\to\infty} \frac{a_i}{b_i} = 0$ and $\sum_{i=1}^{\infty} a_i$ converges

$\Longrightarrow$ the series $\sum_{i=1}^{\infty} b_i$ converges.

$\lim_{i\to\infty} \frac{a_i}{b_i} = \infty$ and $\sum_{i=1}^{\infty} b_i$ diverges

$\Longrightarrow$ the series $\sum_{i=1}^{\infty} a_i$ diverges.

One of the most powerful tests. Squeezes the two series "in the limit".

Alternating Series Test

For the alternating series where all $a_i > 0$

$\sum_{i=1}^{\infty} (-1)^i a_i$
$= a_1 - a_2 + a_3 - a_4 + \cdots$

$a_i \geq a_{i+1}$ for all i and $\lim_{i\to\infty} a_i = 0$

$\Longrightarrow \sum_i^{\infty} (-1)^i a_i$ *converges.*

Absolute Convergence

$\sum_i^{\infty} a_i$ is absolutely convergent

$\Longleftrightarrow$ the sum of absolute values $\sum_i^{\infty} |a_i|$ is convergent.

Ratio Test

$\lim_{i\to\infty} \frac{a_{i+1}}{a_i} = L < 1$

$\Longrightarrow \sum_{i=1}^{\infty} |a_i|$ *converges*

$\Longrightarrow \sum_{i=1}^{\infty} a_i$ *converges.*

$\lim_{i\to\infty} \frac{a_{i+1}}{a_i} = L > 1$

$\Longrightarrow \sum_{i=1}^{\infty} a_i$ *diverges.*

Root Test

$\lim_{n\to\infty} \sqrt[n]{a_n} = L < 1$

$\Longrightarrow \sum_{n=1}^{\infty} |a_n|$ *converges*

$\Longrightarrow \sum_{n=1}^{\infty} a_n$ *converges.*

$\lim_{n\to\infty} \sqrt[n]{a_n} = L > 1$

$\Longrightarrow \sum_{n=1}^{\infty} a_n$ *diverges.*

Integration techniques

u-substitution

Choose a new variable u

Determine the value dx

Make the substitution

Integrate resulting integral

Return to the initial variable x

Integration by parts

Identify u and dv.

Priorities for u are:

$u = \ln x$

$u = x^n$

$u = e^{ax}$

Compute du and v

Use the formula

Partial fractions

The decomposition will be a sum of terms in which the numerators contain coefficients (A, B, C, &c.). The number of these unknown coefficients will always be equal to the degree of the denominator. After the denominator is factored and like terms are collected, we can use the following rules to determine the decomposition.

For a linear term $ax + b$ we get a contribution of $\frac{A}{ax+b}$.

For a repeated linear term, such as $(ax+b)^3$, we get a contribution of

$\frac{A}{ax+b}$
$+ \frac{B}{(ax+b)^2}$
$+ \frac{C}{(ax+b)^3}$

We have *three* terms which match. *ie* $(ax+b)$ to the *third* power.

For a quadratic term $ax^2 + bx + c$ we get a contribution of $\frac{Ax+B}{ax^2+bx+c}$.

For a repeated quadratic $(ax^2+bx+c)^2$ we get a contribution of

$\frac{Ax+B}{ax^2+bx+c}$
$+ \frac{Cx+D}{(ax^2+bx+c)^2}$.

Mixmatch rules if needed.

Trig substitution

$\sqrt{a^2 - x^2} = a\cos\theta$

$x = a\sin\theta$

$dx = a\cos\theta d\theta$

$\sqrt{a^2 + x^2} = a\sec\theta$

$x = a\tan\theta$

$dx = a\sec^2\theta d\theta$

$\sqrt{x^2 - a^2} = a\tan\theta$

$x = a\sec\theta$

$dx = a\sec\theta\tan\theta d\theta$

Improper integrals

If $\int_a^t f(x)dx$ exists for every number $(t \geq a)$,

$\int_a^{\infty} f(x)dx$
$= \lim_{t\to\infty} \int_a^t f(x)dx$

iff lim exists and $\neq \infty$.

If $\int_t^b f(x)dx$ exists for every number $(t \leq b)$,

$\int_{-\infty}^b f(x)dx$
$= \lim_{t\to-\infty} \int_t^b f(x)dx$

iff lim exists and $\neq \infty$.

shit you fogot:

Right Riemann sum

f is approximated by value at right endpoint.
height $f(a + ix)$ rectangles with base x.
Doing this for $i = 0, 1, \cdots, (n-1)$,
and adding up the areas gives
$\Delta x[f(a+\Delta x) + f(a+2\Delta x) + \cdots + f(b)]$.
Underestimation if f is decreasing, over if increasing.
The error will be

$$\left|\int_a^b f(x)\,dx - A_{\text{right}}\right| \leq \frac{M_1(b-a)^2}{2n},$$

M_1 is the maximum value of $\left|f'(x)\right|$ on the interval.

Arc Length Formula

If f' is continuous on $[a,b]$, then the length of the curve $y = f(x)$, $a \leq x \leq b$, is

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

By interchanging the roles of x and y, we obtain the formula

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$$

Surface Area of revolution

$$S = \int 2\pi\rho\,ds$$

where ρ is the axis opposite
to the axis of rotation and

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx \text{ or}$$

$$ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$$

Volume of revolution

The disk method is used when the slice that was drawn is "perpendicular to" the axis of revolution;
i.e. when integrating "parallel to" the axis of revolution.
The volume of the solid formed by rotating the area between the curves of
$f(x)$ and $g(x)$
and the lines $x = a$ and $x = b$
about the x-axis is given by

$$V = \pi\int_a^b \left|f(x)^2 - g(x)^2\right|\,dx.$$

If $g(x) = 0$e.g. revolving an area between the curve and the x-axis, this reduces to:

$$V = \pi\int_a^b f(x)^2\,dx.$$

The method can be visualized by considering a thin horizontal rectangle at y between $f(y)$ on top and $g(y)$ on the bottom, and revolving it about the y-axis; it forms a ring (or disc in the case that $g(y) = 0$), with outer radius $f(y)$ and inner radius g(y).
The area of a ring is $(R^2 r^2)$,
where R is the outer radius (in this case $f(y)$),
and r is the inner radius (in this case g(y)).
The volume of each infinitesimal disc is therefore
$f(y)^2\,dy$. The limit of the Riemann sum of the volumes of the discs between a and b becomes integral (1).

Basic Forms

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln|x|$$

$$\int u\,dv = uv - \int v\,du \text{ (integration by parts)}$$

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b|$$

Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2}\,dx = -\frac{1}{x+a}$$

$$\int (x+a)^n\,dx = \frac{(x+a)^{n+1}}{n+1},$$
$n \neq -1$

$$\int x(x+a)^n\,dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)}$$

$$\int \frac{1}{1+x^2}\,dx = \arctan x$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

$$\int \frac{x}{a^2+x^2}\,dx = \frac{1}{2}\ln|a^2+x^2|$$

$$\int \frac{x^2}{a^2+x^2}\,dx = x - a\arctan\left(\frac{x}{a}\right)$$

$$\int \frac{x^3}{a^2+x^2}\,dx = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln|a^2+x^2|$$

$$\int \frac{1}{ax^2+bx+c}\,dx = \frac{2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

$$\int \frac{1}{(x+a)(x+b)}\,dx = \frac{1}{b-a}\ln\frac{a+x}{b+x},\quad a \neq b$$

$$\int \frac{x}{(x+a)^2}\,dx = \frac{a}{a+x} + \ln|a+x|$$

Integrals with Roots

$$\int \sqrt{x-a}\,dx = \frac{2}{3}(x-a)^{3/2}$$

$$\int \frac{1}{\sqrt{x\pm a}}\,dx = 2\sqrt{x\pm a}$$

$$\int \frac{1}{\sqrt{a-x}}\,dx = -2\sqrt{a-x}$$

$$\int x\sqrt{x-a}\,dx = \frac{2a}{3}(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2},\text{ or}$$

$$\frac{2}{3}x(x-a)^{3/2} - \frac{4}{15}(x-a)^{5/2},\text{ or}$$

$$\frac{2}{15}(2a+3x)(x-a)^{3/2}$$

$$\int \sqrt{ax+b}\,dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{ax+b}$$

$$\int (ax+b)^{3/2}\,dx = \frac{2}{5a}(ax+b)^{5/2}$$

$$\int \frac{x}{\sqrt{x\pm a}}\,dx = \frac{2}{3}(x\mp 2a)\sqrt{x\pm a}$$

$$\int \sqrt{\frac{x}{a-x}}\,dx = -\sqrt{x(a-x)} - a\arctan\left(\frac{\sqrt{x(a-x)}}{x-a}\right)$$

$$\int \sqrt{\frac{x}{a+x}}\,dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x} + \sqrt{x+a}\right]$$

$$\int x\sqrt{ax+b}\,dx = \frac{2(-2b^2+abx+3a^2x^2)\sqrt{ax+b}}{15a^2}$$

$$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \pm \frac{1}{2}a^2\ln\left|x+\sqrt{x^2\pm a^2}\right|$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)}{2}$$

$$\int x\sqrt{x^2\pm a^2}\,dx = \frac{1}{3}\left(x^2\pm a^2\right)^{3/2}$$

$$\int \frac{1}{\sqrt{x^2\pm a^2}}\,dx = \ln\left|x+\sqrt{x^2\pm a^2}\right|$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\frac{x}{a}$$

$$\int \frac{x}{\sqrt{x^2\pm a^2}}\,dx = \sqrt{x^2\pm a^2}$$

$$\int \frac{x}{\sqrt{a^2-x^2}}\,dx = -\sqrt{a^2-x^2}$$

$$\int \frac{x^2}{\sqrt{x^2\pm a^2}}\,dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \mp \frac{1}{2}a^2\ln\left|x+\sqrt{x^2\pm a^2}\right|$$

$$\int \sqrt{ax^2+bx+c}\,dx = \frac{b+2ax}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a^{3/2}} \times \ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right|$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}}\,dx = \frac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right|$$

$$\int \frac{x}{\sqrt{ax^2+bx+c}}\,dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2a^{3/2}} \times \ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right|$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}$$

Integrals with Logarithms

$$\int \ln ax\,dx = x\ln ax - x$$

$$\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \frac{x^2}{4}$$

$$\int x^2\ln x\,dx = \frac{1}{3}x^3\ln x - \frac{x^3}{9}$$

$$\int x^n\ln x\,dx = x^{n+1} \times \left(\frac{\ln x}{n+1} - \frac{1}{(n+1)^2}\right),\quad n \neq -1$$

$$\int \frac{\ln ax}{x}\,dx = \frac{1}{2}(\ln ax)^2$$

$$\int \frac{\ln x}{x^2}\,dx = -\frac{1}{x} - \frac{\ln x}{x}$$

$$\int \ln(ax+b)\,dx = \left(x + \frac{b}{a}\right)\ln(ax+b) - x,\quad a \neq 0$$

$$\int \ln(x^2+a^2)\,dx = x\ln(x^2+a^2) + 2a\tan^{-1}\frac{x}{a} - 2x$$

$$\int \ln(x^2-a^2)\,dx = x\ln(x^2-a^2) + a\ln\frac{x+a}{x-a} - 2x$$

$$\int \ln\left(ax^2+bx+c\right)dx = \frac{1}{a}\sqrt{4ac-b^2}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + \left(\frac{b}{2a}+x\right)\ln\left(ax^2+bx+c\right)$$

$$\int x\ln(ax+b)\,dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right)\ln(ax+b)$$

$$\int x\ln\left(a^2-b^2x^2\right)dx = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 - \frac{a^2}{b^2}\right)\ln\left(a^2-b^2x^2\right)$$

$$\int (\ln x)^2\,dx = 2x - 2x\ln x + x(\ln x)^2$$

$$\int (\ln x)^3\,dx = -6x + x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x$$

$$\int x(\ln x)^2\,dx = \frac{x^2}{4} + \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x$$

$$\int x^2(\ln x)^2\,dx = \frac{2x^3}{27} + \frac{1}{3}x^3(\ln x)^2 - \frac{2}{9}x^3\ln x$$

Integrals with Exponentials

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \sqrt{x}e^{ax}\,dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}} \times \operatorname{erf}\left(i\sqrt{ax}\right),\text{ where } \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$$

$$\int xe^x\,dx = (x-1)e^x$$

$$\int xe^{ax}\,dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax}$$

$$\int x^2e^x\,dx = \left(x^2-2x+2\right)e^x$$

$$\int x^2e^{ax}\,dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)e^{ax}$$

$$\int x^3e^x\,dx = \left(x^3-3x^2+6x-6\right)e^x$$

$$\int x^ne^{ax}\,dx = \frac{x^ne^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}dx$$

$$\int x^ne^{ax}\,dx = \frac{(-1)^n}{a^{n+1}}\Gamma[1+n,-ax],$$

where $\Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}dt$

$$\int xe^{-ax^2}\,dx = -\frac{1}{2a}e^{-ax^2}$$

Integrals with Trigonometric Functions

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int \sin^3 ax\,dx = -\frac{3\cos ax}{4a} + \frac{\cos 3ax}{12a}$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int \cos^3 ax\,dx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a}$$

$$\int \cos x\sin x\,dx = \frac{1}{2}\sin^2 x + c_1 = -\frac{1}{2}\cos^2 x + c_2 = -\frac{1}{4}\cos 2x + c_3$$

$$\int \cos ax\sin bx\,dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)},\quad a \neq b$$

$$\int \sin^2 ax\cos bx\,dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)}$$

$$\int \sin^2 x\cos x\,dx = \frac{1}{3}\sin^3 x$$

$$\int \cos^2 ax\sin bx\,dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)}$$

$$\int \cos^2 ax\sin ax\,dx = -\frac{1}{3a}\cos^3 ax$$

$$\int \sin^2 ax\cos^2 bx\,dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)}$$

$$\int \sin^2 ax\cos^2 ax\,dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$$

$$\int \tan ax\,dx = -\frac{1}{a}\ln\cos ax$$

$$\int \tan^2 ax\,dx = -x + \frac{1}{a}\tan ax$$

$$\int \tan^3 ax\,dx = \frac{1}{a}\ln\cos ax + \frac{1}{2a}\sec^2 ax$$

$$\int \sec x\,dx = \ln|\sec x + \tan x|$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax$$

$$\int \sec^3 x\,dx = \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x + \tan x|$$

$$\int \sec x\tan x\,dx = \sec x$$

$$\int \sec^2 x\tan x\,dx = \frac{1}{2}\sec^2 x$$

$$\int \sec^n x\tan x\,dx = \frac{1}{n}\sec^n x,\quad n \neq 0$$

$$\int \csc x\,dx = \ln\left|\tan\frac{x}{2}\right| = \ln|\csc x - \cot x| + C$$

$$\int \csc^2 ax\,dx = -\frac{1}{a}\cot ax$$

$$\int \csc^3 x\,dx = -\frac{1}{2}\cot x\csc x + \frac{1}{2}\ln|\csc x - \cot x|$$

$$\int \csc^n x\cot x\,dx = -\frac{1}{n}\csc^n x,\quad n \neq 0$$

$$\int \sec x\csc x\,dx = \ln|\tan x|$$

Products of Trigonometric Functions and Monomials

$$\int x\cos x\,dx = \cos x + x\sin x$$

$$\int x\cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$$

$$\int x^2\cos x\,dx = 2x\cos x + \left(x^2-2\right)\sin x$$

$$\int x^2\cos ax\,dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2-2}{a^3}\sin ax$$

$$\int x\sin x\,dx = -x\cos x + \sin x$$

$$\int x\sin ax\,dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2}$$

$$\int x^2\sin x\,dx = \left(2-x^2\right)\cos x + 2x\sin x$$

$$\int x^2\sin ax\,dx = \frac{2-a^2x^2}{a^3}\cos ax + \frac{2x\sin ax}{a^2}$$

$$\int x\cos^2 x\,dx = \frac{x^2}{4} + \frac{1}{8}\cos 2x + \frac{1}{4}x\sin 2x$$

$$\int x\sin^2 x\,dx = \frac{x^2}{4} - \frac{1}{8}\cos 2x - \frac{1}{4}x\sin 2x$$

$$\int x\tan^2 x\,dx = -\frac{x^2}{2} + \ln\cos x + x\tan x$$

$$\int x\sec^2 x\,dx = \ln\cos x + x\tan x$$

Products of Trigonometric Functions and Exponentials

$$\int e^x\sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x)$$

$$\int e^{bx}\sin ax\,dx = \frac{1}{a^2+b^2}e^{bx}(b\sin ax - a\cos ax)$$

$$\int e^x\cos x\,dx = \frac{1}{2}e^x(\sin x + \cos x)$$

$$\int e^{bx}\cos ax\,dx = \frac{1}{a^2+b^2}e^{bx}(a\sin ax + b\cos ax)$$

$$\int xe^x\sin x\,dx = \frac{1}{2}e^x \times (\cos x - x\cos x + x\sin x)$$

$$\int xe^x\cos x\,dx = \frac{1}{2}e^x \times (x\cos x - \sin x + x\sin x)$$