Mathematics 1120H – Calculus II: Integrals and Series
TRENT UNIVERSITY, Winter 2022

Assignment #10
A Power Series For $\mathbf{e^x}$
Due on Friday, 1 April.
(May be submitted on paper or via Blackboard.)*

Please show all your work. As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

In what follows, let $f(x) = e^x$ and let $g(x) = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$.

1. Determine for which values of x the series defining $g(x)$ converges conditionally, converges absolutely, or diverges. *[4]*

In question **2**, you may assume that one can differentiate power series term-by-term for those values of x for which the power series converges.

2. Show that both $y = f(x)$ and $y = g(x)$ satisfy the differential equation $\dfrac{dy}{dx} = y$. *[2]*

3. Use your answer to **2** to help conclude that $f(x) = g(x)$. *[2]*

For question **4**, you may assume that, for all $x \in \mathbb{R}$, $\cos(x) = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$ and $\sin(x) = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$. In addition to this you may assume that i is the square root of -1, *i.e.* $i^2 = -1$.

4. Use what you showed in answering question **3**, plus the information above, to prove *Euler's Formula*: $e^{i\theta} = \cos(\theta) + i\sin(\theta)$. *[2]*

NOTE. Plugging in $\theta = \pi$ into Euler's Formula gives the equation $e^{i\pi} = -1$, which is also sometimes called Euler's Formula.

* All else failing, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`