Mathematics 1110H – Calculus I: Limits, derivatives, and Integrals

TRENT UNIVERSITY, Winter 2021

Solutions to Quiz #2

Tuesday, 26 January.

Do *both* of the following questions. Show all your work!

1. Compute $\lim_{x\to 4} \dfrac{x^2-x-12}{x^2+x-20}$. *[2.5]*

SOLUTION. Both the numerator and the denominator approach 0 as x approaches 4, so we seek to simplify them to evaluate the limit. The key is to factor them both:

$$\lim_{x\to 4} \frac{x^2-x-12}{x^2+x-20} = \lim_{x\to 4} \frac{(x-4)(x+3)}{(x-4)(x+5)} = \lim_{x\to 4} \frac{x+3}{x+5} = \frac{4+3}{4+5} = \frac{7}{9} \qquad \square$$

2. Compute $\lim_{x\to \pi/4} \dfrac{1-\sin(2x)}{\sin(x)-\cos(x)}$. *[2.5]*

SOLUTION. Again, both the numerator and the denominator approach 0 as x approaches 4, so we seek to simplify them to evaluate the limit. The key is to expand the numerator using the trigonometric identities $\sin(2x) = 2\sin(x)\cos(x)$ and $\sin^2(x)+\cos^2(x) = 1$, and then factor it:

$$\begin{aligned}
\lim_{x\to \pi/4} \frac{1-\sin(2x)}{\sin(x)-\cos(x)} &= \lim_{x\to \pi/4} \frac{\sin^2(x)+\cos^2(x)-2\sin(x)\cos(x)}{\sin(x)-\cos(x)} \\
&= \lim_{x\to \pi/4} \frac{\sin^2(x)-2\sin(x)\cos(x)+\cos^2(x)}{\sin(x)-\cos(x)} \\
&= \lim_{x\to \pi/4} \frac{(\sin(x)-\cos(x))^2}{\sin(x)-\cos(x)} \\
&= \lim_{x\to \pi/4} (\sin(x)-\cos(x)) \\
&= \sin\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0 \qquad \blacksquare
\end{aligned}$$

[Total = 5]