Mathematics 1120H – Calculus II: Integrals and Series
Trent University, Winter 2020

Assignment #5.1 – the `Maple`-less edition
Guaranteeing Convergence
Due on Thursday, 26 March.

Instructions: *You may do one of Assignment #5.1 and the original Assignment #5. Either way, please submit your solutions on or by the due date using the assignment submission tool on Blackboard, preferably as a pdf. If that doesn't work, please email it to your instructor.*

It's a fact that $\frac{1}{e} = e^{-1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = 1 - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120} - \cdots$. (We'll see why this series adds up to $\frac{1}{e}$ once we do Taylor series.)

1. Find a value of m such that $\sum_{n=0}^{k} \frac{(-1)^n}{n!} = 1 - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \cdots + \frac{(-1)^k}{k!}$ is guaranteed to be within $0.0001 = 10^{-4}$ of $\frac{1}{e}$ for all $k \geq m$ and explain why it's guaranteed. *[3]*

It's also a fact that $\frac{\pi}{4} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$. (Again, we'll see why this series adds up to $\frac{\pi}{4}$ once we do Taylor series.)

2. Find a value of m such that $\sum_{n=0}^{k} \frac{(-1)^n}{2n+1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots + \frac{(-1)^k}{2k+1}$ is guaranteed to be within $0.0001 = 10^{-4}$ of $\frac{\pi}{4}$ for all $k \geq m$ and explain why it's guaranteed. *[3]*

The series $\sum_{n=0}^{\infty} \frac{2}{(4n+1)(4n+3)}$ also adds up to $\frac{\pi}{4}$. (To see why, do a little algebra to answer question **3**.)

3. How are these two series adding up to $\frac{\pi}{4}$ related, besides having the same sum? *[4]*