

Mathematics 1120H – Calculus II: Integrals and Series

TRENT UNIVERSITY, Winter 2020

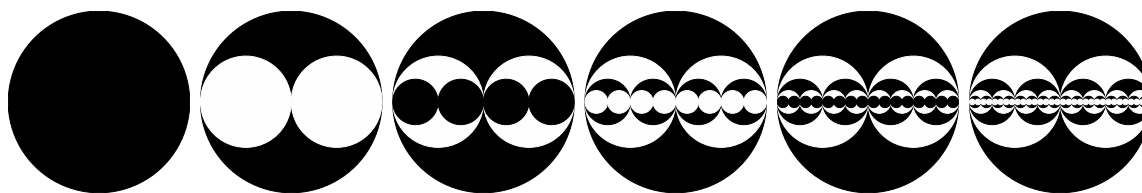
Assignment #4

Alien Eyes?

Due on Thursday, 5 March.

Consider the following process. At step 0, we start with a solid disk of radius 1. At step 1, we remove two disks of radius  $\frac{1}{2}$ , which are tangent to each other and to the perimeter of the original disk, from the solid disk. At step 2, we insert two solid disks of radius  $\frac{1}{4}$ , which are tangent to each other and to the perimeter of the removed disk, into each of the removed disks. At step 3, we remove two disks of radius  $\frac{1}{8}$ , which are tangent to each other and to the perimeter of the solid disk added at the previous step, from each of the solid disk added at the previous step. In general, at each odd step  $n$  we remove two disks of radius  $(\frac{1}{2})^n$  from each of the solid disks we added at the previous step, and at each even step  $n$  we insert two solid disks of radius  $(\frac{1}{2})^n$  into each of the disks removed at the previous step.

The shapes at steps 0 through 5 are pictured below:



1. What is the total length of the perimeter of the (solid) shape at stage  $n \geq 0$ ? [3]
2. What is the total area of the (solid) shape at stage  $n \geq 0$ ? [4]
3. What is the total length of the perimeter of the (solid) shape one would have after all infinitely many steps of this process have been done? [1]
4. What is the total area of the perimeter of the (solid) shape one would have after all infinitely many steps of this process have been done? [2]