Mathematics 1120H – Calculus II: Integrals and Series
Trent University, Winter 2019
Final Examination
19:00-22:00 on Monday, 22 April, in the Gym.

Time: *3 hours.* *Brought to you by* Стефан Біланюк.

Instructions: Do parts **X**, **Y**, and **Z**, and, if you wish, part **W**. Show all your work and justify all your answers. *If in doubt about something,* **ask!**

Aids: Any calculator; (all sides of) one aid sheet; one (1) brain (no neuron limit).

Part X. Do all four (4) of **1**–**4**.

1. Evaluate any *four* (4) of the integrals **a**–**f**. *[20 = 4 × 5 each]*

a. $\displaystyle\int_{-1}^{1} \frac{1}{(x+2)^2}\,dx$ **b.** $\displaystyle\int z\cos(z)\,dz$ **c.** $\displaystyle\int \left(1-y^2\right)^{-1/2}\,dy$

d. $\displaystyle\int_{-\infty}^{0} 2ue^{u^2}\,du$ **e.** $\displaystyle\int \frac{1}{v^3+v}\,dv$ **f.** $\displaystyle\int_{0}^{\pi/2} \frac{\cos(w)}{\sin^2(w)+1}\,dw$

2. Determine whether the series converges in any *four* (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{3^n+4^n}$ **b.** $\displaystyle\sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{\ln(m+2)}$ **c.** $\displaystyle\sum_{\ell=2}^{\infty} \frac{e^{\ell}}{e^{2\ell}+1}$

d. $\displaystyle\sum_{k=3}^{\infty} \frac{(-1)^k 17^k}{k^k}$ **e.** $\displaystyle\sum_{j=4}^{\infty} \frac{j!}{(2j)!}$ **f.** $\displaystyle\sum_{i=5}^{\infty} \frac{(i+1)^3}{(i+2)^5}$

3. Do any *four* (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. Use the Right-Hand Rule to compute $\displaystyle\int_0^2 (x+1)\,dx$.

b. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converges or diverges.

c. Find the area of the region between $y=\sqrt{x+1}$ and $y=\dfrac{x}{3}+1$, where $0 \le x \le 3$.

d. Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} 2^{n+1}x^n$.

e. Compute $\displaystyle\lim_{n\to\infty} \frac{2^n}{n!}$. *[Hint: Squeeze!]*

f. Find the volume of the solid obtained by revolving the region between $y=x$ and $y=0$, where $0 \le x \le 1$, about the x-axis.

4. Find the centroid of the region below $y=\sqrt{4-x^2}$ and above $y=0$, where $0 \le x \le 2$. [You may assume that the density is constant and units have been chosen so that *mass = area*.] *[12]*

Part Y. Do either *one* (1) of **5** or **6**. *[14]*

5. Consider the curve $y = \frac{2}{3}x^{3/2}$, where $0 \le x \le 3$.

a. Find the arc-length of the curve. *[7]*

b. Find the area of the surface obtained by revolving the curve about the y-axis. *[7]*

6. Consider the triangle whose vertices are the points $(0,0)$, $(1,1)$, and $(2,0)$. Find the volume of the solid obtained by revolving this triangle about ...

a. ... the x-axis. *[7]*

b. ... the y-axis. *[7]*

Part Z. Do either *one* (1) of **7** or **8**. *[14]*

7. Find the Taylor series at $a = 0$ of $f(x) = \frac{1}{1+2x}$ and determine its radius and interval of convergence.

8. **a.** Use Taylor's formula to find the Taylor series at $a = 0$ of $g(x) = e^x$ and determine its radius and interval of convergence. *[10]*

b. How many terms of this Taylor series are needed to guarantee that if the partial sum is evaluated at $x = 1$, it will be within $0.01 = \frac{1}{100}$ of $g(1) = e^1 = e$? *[4]*

[Total = 100]

Part W. Bonus problems! If you feel like it and have the time, do one or both of these.

W. Consider the following real number:

$$a = \sum_{n=0}^{\infty} \frac{1}{10^{[2^n]}} = \sum_{n=0}^{\infty} 10^{-[2^n]} = 0.11010001000000010\cdots$$

[For $k \ge 1$, there are $2^{k-1} - 1$ zeros between the kth and $(k+1)$st ones in the decimal expansion of a.] Explain why a must be irrational. *[1]*

M. Write a haiku (or several :-) touching on calculus or mathematics in general. *[1]*

What is a haiku?

seventeen in three:
five and seven and five of
syllables in lines

ENJOY YOUR SUMMER!

P.S.: You can keep this question sheet. Solutions to this exam will be posted to the course archive page at `euclid.trentu.ca/math/sb/1120H/` *in a day or two.*