Mathematics 1120H – Calculus II: Integrals and Series
TRENT UNIVERSITY, Summer 2018

Assignment #2π
A sequence of roots
Due at the exam on Monday, 30 July.

Consider the sequence defined by $a_0 = 0$ and, for all $n \geq 0$, $a_{n+1} = \sqrt{2 + a_n}$.

1. Show that this sequence has a limit. *[5]*

HINT. Show that the sequence is bounded above by 2, *i.e.* $a_n < 2$ for all $n \geq 0$, and is increasing, *i.e.* $a_n < a_{n+1}$ for all $n \geq 0$, and then apply the Monotone Convergence Theorem (Theorem 11.1.12 in §11.1 of the textbook).

2. Compute $\lim_{n \to \infty} a_n$. *[5]*

HINT. Take the limit of both sides of the equation $a_{n+1} = \sqrt{2 + a_n}$ that was used to help define the sequence.

Bonus. What is the value of the infinite product

$$\prod_{n=1}^{\infty} \frac{2}{a_n} = \frac{2}{\sqrt{2}} \cdot \frac{2}{\sqrt{2+\sqrt{2}}} \cdot \frac{2}{\sqrt{2+\sqrt{2+\sqrt{2}}}} \cdots \quad ?$$

Explain why. *[3]*

HINT. The number $4 \cdot \prod_{n=1}^{\infty} \frac{2}{a_n}$ appears on this page.