Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals

TRENT UNIVERSITY, Summer 2023 (S61)

Assignment #5
Right-Hand Rule

Due just before midnight on Friday, 9 June.*

Recall from class that the Right-Hand Rule for computing the definite integral of $f(x)$, *i.e.* weighted area between $y = f(x)$ and the x-axis, for x between a and b, is:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \left[\sum_{i=1}^{n} \frac{b-a}{n} \cdot f\left(a + i \cdot \frac{b-a}{n}\right) \right]$$

This actually works as a definition of the definite integral when $f(x)$ is nice enough, such as when it is continuous on $[a, b]$, but even then some basic properties of definite integrals are hard to prove. As a computational method for calculating definite integrals, it's not very useful because even simple integrals can take a while to work through. (See the example we did in class.) In this assignment, you will be asked to do so anyway ... :-)

1. Use the Right-Hand Rule to compute $\displaystyle\int_1^4 \left(x^2+1\right) dx$ by hand. *[6]*

You may find the summation formulas $\displaystyle\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$ and $\displaystyle\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ to be useful in working through **1**.

2. Use the Right-Hand Rule to compute $\displaystyle\int_1^4 \left(x^2+1\right) dx$ using SageMath. *[4]*

You may find SageMath's `sum` command, introduced in the lab of 2023-05-31, and `limit` command, introduced in the lab of 2023-05-10, to be of use in working through **2**.

* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If this fails, you may submit your work to the instructor on paper or by email to `sbilaniuk@ trentu.ca` .