Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Summer 2023 (S61)

Assignment #2
Solving equations with and without SageMath
Due just before midnight on Friday, 19 May.*

Before tackling this assignment, take a peek at the file `1110H-lab-20230510.pdf`, which you can find in the Labs folder in the Course Content section on Blackboard. Skimming and later referring to as necessary to Sections 1.8 and 4.8 of Gregory Bard's book *Sage for Undergraduates* (in the SageMath folder in the Course Content section on Blackboard) is probably a good idea. If you wish to use another general purpose mathematics application, such as Maple or Mathematica, you may, but you're on your own for learning to use it and getting help.

1. The Indian mathematician Bhaskara (1114-1185 A.D.), often referred to as Bhaskara II to distinguish him from an earlier mathematician named Bhaskara (*c.* 600-680 A.D.), posed the following problem in a book dedicated to his daughter Lilavati:

> *The square root of half the number of bees in a swarm has flown out upon a jasmine bush. Eight-ninths of the swarm has remained behind, and a female bee flies about a male who is buzzing inside a lotus flower; in the night, attracted by the flower's sweet odour, he went inside it, and now he is trapped! Tell me, most enchanting lady, the number of bees.*

The original text is actually in verse written in Sanskrit; the above is something of a loose translation. For those interested in the history of mathematics, Bhaskara developed a number of techniques that anticipated portions of both differential and integral calculus.

a. Express the information given in the problem as an equation. *[1]*
b. Solve the equation you obtained by hand. *[1]*
c. Solve the equation you obtained using SageMath. *[1]*
d. *Bonus!* What does this problem have to do with a Monty Python sketch? *[0.5]*

2. The *hyperbolic functions* include:

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{csch}(x) = \frac{1}{\sinh(x)} \qquad \operatorname{sech}(x) = \frac{1}{\cosh(x)} \qquad \coth(x) = \frac{\cosh(x)}{\sinh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

The names of these function are usually pronounced something like "sinch", "kosh", "tanch", "co-seech", "seech", and "kotch", respectively. They turn out to be closely related to the natural exponential function (obviously) and the trigonometric functions; the latter connections being more obvious when you look at their series expansions and expecially when you start looking at them as functions of a complex variable.

* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If this fails, you may submit your work to the instructor on paper or by email to `sbilaniuk@ trentu.ca` .

a. Use SageMath to compute $\lim_{x \to -\infty} \text{sech}(x)$. *[1]*

b. Find a formula for the inverse function, arccsinh(x), of sinh(x) by hand. *[3]*

c. Find a formula for the inverse function of sinh(x) using SageMath. *[3]*

NOTE: Recall that a function $f(x)$ is the inverse of a function $g(x)$ if it undoes what $g(x)$ does, *i.e.* $f(g(x)) = x$. To put it another way, if $y = g(x)$, then $x = f(y)$, and *vice versa*.

Hint: arcsinh(x) is a natural logarithm of a quadratic expression. For fun and practice, try to figure out what its domain is once you're figured out what it is.